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ORIGINAL ARTICLE

Note on the single-shock solutions of the Korteweg-de Vries-Burgers equation

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Abstract The well-known shock solutions of the Korteweg-de Vries-Burgers equation are revisited, together with their limitations in the context of plasma (astro)physical applications. Although available in the literature for a long time, it seems to have been forgotten in recent papers that such shocks are monotonic and unique, for a given plasma configuration, and cannot show oscillatory or bell-shaped features. This uniqueness is contrasted to solitary wave solutions of the two parent equations (Korteweg-de Vries and Burgers), which form a family of curves parameterized by the excess velocity over the linear phase speed.

Keywords Plasmas · Shock waves

Among the paradigm nonlinear evolution equations cropping up in various domains of physics, the Korteweg-de Vries-Burgers (KdVB) equation,

$$\frac{\partial \varphi_1}{\partial \tau} + A\varphi_1 \frac{\partial \varphi_1}{\partial \xi} + B\frac{\partial^3 \varphi_1}{\partial \xi^3} = C\frac{\partial^2 \varphi_1}{\partial \xi^2}, \tag{1}$$

arises in physical media where nonlinearity, dispersion and damping interact on slow timescales to produce solitary structures. More specifically, in plasma physics (1) typically obtains by reductive perturbation analysis of a multi-fluid model, through the use of coordinate stretching

$$\xi = \varepsilon^{1/2}(x - \lambda t), \qquad \tau = \varepsilon^{3/2} t, \tag{2}$$

combined with expansions of the dependent variables like

$$\varphi = \varepsilon\varphi_1 + \varepsilon^2\varphi_2 + \cdots \tag{3}$$

in addition to an appropriate scaling of the damping coefficient, in many cases due to viscosity. Here x and t are the original space and time coordinates, respectively, and φ refers to the electrostatic potential of the solitary waves. In the absence of damping ($C = 0$), the KdVB equation (1) reduces to the KdV equation, whereas in the absence of dispersion ($B = 0$), it recovers the Burgers equation, which bears kink-shaped monotonic shock profile solutions. All this is well known and has been in the literature for a long time, but we will have to come back to these points later.

For a purely mathematical study of the properties of the KdVB equation, (1) is given and its coefficients A, B and C might be regarded as free parameters. However, the moment the KdVB equation is derived for a particular plasma (astro)physical configuration, the precise and often elaborate form of A, B and C has to be computed. Although the intermediate details need not concern us here, we still have to remind ourselves that A, B and C are functions of the plasma compositional parameters, which also determine the linear phase velocity λ, and thus cannot be chosen randomly. Moreover, in the process of deriving (1) one has imposed/used that φ_1 vanishes in the undisturbed medium, upstream of the shock or soliton solutions, translated as $\varphi_1 \to 0$ for $\xi \to +\infty$. All this has important consequences for the discussion which follows.

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