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ORIGINAL ARTICLE

Computation of families of periodic orbits and bifurcations around a massive annulus

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Abstract This paper studies the main features of the dynamics around a planar annular disk. It is addressed an appropriated closed expression of the gravitational potential of a massive disk, which overcomes the difficulties found in previous works in this matter concerning its numerical treatment. This allows us to define the differential equations of motion that describes the motion of a massless particle orbiting the annulus. We describe the computation methods proposed for the continuation of uni-parametric families of periodic orbits, these algorithms have been applied to analyze the dynamics around a massive annulus by means of a description of the main families of periodic orbits found, their bifurcations and linear stability.

Keywords Periodic orbits · Bifurcation of families · Solid annulus disk potential

1 Introduction

Outer planets of the Solar System and probably many of the Extrasolar ones have rings. Scientific exploration missions aimed to study the boundaries of the solar System, such as the space probes Pioner11 (1979), Voyager1 (1980), Voyager2 (1981) or the most recent and relevant program of planetary exploration Cassini-Huygens (2004), have provided an in-depth knowledge of the planetary ring systems, determining the structure, composition and dynamical behavior of the planet, rings and moons. Planetary rings are made of millions of rocky and icy particles, each maintaining their own orbit around the planet inside its Roche limit; these small orbiting particles can be considered, from a distance, as a continuous solid annular ring. Ring systems can be also found from disks around supermassive black holes to protoplanetary disks that give rise to planets, the Kuiper belt for example is the remnant of the disk that rotated around the Sun.

Many authors studied the dynamics of planetary ring systems. One of the pioneers, Maxwell (1859) proposed a model for the motion of the particles surrounding Saturn considering a polygonal configuration for the planar $(n+1)$ body problem, in such a way that n bodies of equal mass are located at the vertices of a regular n-gon centered at the remaining body. This model attracted the interest of researchers (Scheeres 1992; Breiter et al. 1996; Kalvouridis 1999; Arribas and Elipe 2005; Arribas et al. 2007; Alberti and Vidal 2007; Elipe et al. 2007) in the last years because of the possibility of considering this type of configuration for different dynamical systems. The dynamical models proposed consider a gravitation potential created by a ring, where the ring is described as a finite number of particles placed in a ring configuration, or a solid circular wire.

On the other hand we find authors, such as Stone and Balbus (1996) or Tiscareno et al. (2007) working on the dynamics of disk formation, angular momentum transport, density waves or disk instability and mass transfer; or such as Longaretti (1989), Sicardy (1991) or Benet and Merlo (2009) that carried out different studies based on the data observed

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