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ORIGINAL ARTICLE

Newtonian and post Newtonian expansionfree fluid evolution in $f(R)$ gravity

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Abstract We consider a collapsing sphere and discuss its evolution under the vanishing expansion scalar in the framework of $f(R)$ gravity. The fluid is assumed to be locally anisotropic which evolves adiabatically. To study the dynamics of the collapsing fluid, Newtonian and post Newtonian regimes are taken into account. The field equations are investigated for a well-known $f(R)$ model of the form $R+\delta R^2$ admitting Schwarzschild solution. The perturbation scheme is used on the dynamical equations to explore the instability conditions of expansionfree fluid evolution. We conclude that instability conditions depend upon pressure anisotropy, energy density and some constraints arising from this theory.

Keywords $f(R)$ gravity · Instability · Newtonian and post Newtonian regimes

1 Introduction

To accommodate observational data with theoretical predictions, one needs to introduce the dark energy (DE) contributions in the most successful gravitational theory of General Relativity (GR). On the other hand, modified theories of gravity may provide a cosmological accelerating mechanism without introducing any extra DE contribution. Modifications of GR by $f(R)$ models are able to mimic the standard ΛCDM cosmological evolution. These models may have vacuum solutions with null scalar curvature that allow to recover some GR solutions. These may also lead to the existence of some new solutions particularly in spherically symmetric scenario.

In the last decade, some $f(R)$ models were considered to modify GR at small scales to explain inflation, e.g., $f(R)\propto R^2$ but failed to explain the late time acceleration. A model like $f(R)\propto \frac{1}{R}$ was proposed to explain this acceleration but attained no interest due to conflict with solar system tests (Chiba 2003; Dolgov and Kawasaki 2003). A cosmological viable model needs to satisfy the evolution of big-bang nucleosynthesis, radiation and matter dominated eras. Also, they must provide cosmological perturbations compatible with cosmological constraints from cosmic microwave background and large scale structure. The problem of cosmological perturbations in this modified theory and their consequences have widely been discussed in literature (e.g., Hwang and Noh 2006; Carroll et al. 2006; Carloni et al. 2008; Tsujikawa et al. 2008). Since the Lagrangian $R+f(R)$ is analytic at $R=0$, so the Schwarzschild and other important GR solutions (without cosmological constant) are also the solutions of $f(R)$ gravity (de la Cruz-Dombrize and Dobado 2006).

It is known that $f(R)$ models reveal black hole solutions as in GR, therefore it is quite natural to discuss the question of black hole features and dynamics of the gravitational collapse in this modified theory. When we take conformal transformation of $f(R)$ action, it is found that Schwarzschild solution is the only static spherically symmetric solution for a model of the form $R+\delta R^2$ (Whitt 1984). Also, the uniqueness theorems of spherically symmetric solutions for general polynomial action in arbitrary dimension were proposed by using conformal transformation (Mignemi and Wiltshire 1992). Work on black hole solutions in $f(R)$ gravity has been carried out by many authors, e.g., (Moon et al. 2011;

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