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ORIGINAL ARTICLE

Strong and weak gravitational field in $R+\mu^4/R$ gravity

K. Saaidi · A. Vajdi · S.W. Rabiei ·
A. Aghamohammadi · H. Sheikhahmadi

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Abstract We introduce a new approach for investigating the weak field limit of vacuum field equations in $f(R)$ gravity and we find the weak field limit of $f(R)=R+\mu^4/R$ gravity. Furthermore, we study the strong gravity regime in $R+\mu^4/R$ model of $f(R)$ gravity. We show the existence of strong gravitational field in vacuum for such model. We find out in the limit $\mu\to 0$, the weak field limit and the strong gravitational field can be regarded as a perturbed Schwarzschild metric.

Keywords Spherically symmetric solution · $f(R)$ gravity · General relativity

1 Introductions

Observations on supernova type Ia (Riess et al. 1998; Perlmutter et al. 1999), cosmic microwave background (Spergel et al. 2003) and large scale structure (Tegmark et al. 2004), all indicate that the expansion of the universe is not proceeding as predicted by general relativity, if the universe is homogeneous, spatially flat, and filled with relativistic matter. An interesting approach to explain the positive acceleration of the universe is $f(R)$ theories of gravity which generalize the geometrical part of Hilbert–Einstein Lagrangian (Capozziello 2002; Carroll et al. 2004, 2005; Clifton and Barrow 2005; Nojiri and Odintsov 2003; Sawicki and Hu 2007b; Evans et al. 2008; Aghmohammadi et al. 2009). One of the initiative $f(R)$ models supposed to explain the positive acceleration of expanding universe has $f(R)$ action as $f(R)=R-\mu^4/R$ (Carroll et al. 2004). After proposing the $f(R)=R-\mu^4/R$ model, it was appeared this model suffer several problems. In the metric formalism, initially Dolgov and Kawasaki discovered the violent instability in the matter sector (Dolgov and Kawasaki 2003). The analysis of this instability generalized to arbitrary $f(R)$ models (Faraoni 2006; Sawicki and Hu 2007a) and it was shown than an $f(R)$ model is stable if $d^2f/dR^2>0$ and unstable if $d^2f/dR^2<0$. Thus we can deduce $R-\mu^4/R$ suffer the Dolgov–Kawasaki instability but this instability removes in the $R+\mu^4/R$ model, where $\mu^4>0$. Furthermore, one can see in the $R-\mu^4/R$ model the cosmology is inconsistent with observation when non-relativistic matter is present. In fact there is no matter dominant era (Amendola et al. 2007a, 2007b; Evans et al. 2008). However, the recent study shows the standard epoch of matter domination can be obtained in the $R+\mu^4/R$ model (Evans et al. 2008).

It is obvious that a viable theory of gravity must have the correct Newtonian limit. Indeed a viable theory of $f(R)$ gravity must pass solar system tests. After the $R-\mu^4/R$ was suggested as the solution of cosmic-acceleration puzzle, it has been argued that this theory is inconsistent with solar system tests (Chiba 2003). This claim was based on the fact that metric $f(R)$ gravity is equivalent to $\omega=0$ Brans–Dicke theory, while the observational constraint is $\omega>40000$. But this is not quite the case and it is possible to investigate the spherical symmetric solutions of $f(R)$ gravity without invoking the equivalence of $f(R)$ gravity and scalar tensor theory (Clifton and Barrow 2005; Cembranos 2006; Sawicki and Hu 2007b; Multamaki and Vilja 2006; Capozziello and Stabile 2009; Capozziello et al. 2008, 2010;

K. Saaidi (✉) · A. Vajdi · S.W. Rabiei · H. Sheikhahmadi
Department of Physics, Faculty of Science, University of Kurdistan, Sanandaj, Iran
e-mail: ksaaidi@uok.ac.ir

A. Aghamohammadi
Department of Physics, Sanandaj Branch, Islamic Azad University, Sanandaj, Iran