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ORIGINAL ARTICLE

Non-linear stability in photogravitational non-planar restricted three body problem with oblate smaller primary

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Abstract We have discussed non-linear stability in photogravitational non-planar restricted three body problem with oblate smaller primary. By photogravitational we mean that both primaries are radiating. We normalized the Hamiltonian using Lie transform as in Coppola and Rand (Celest. Mech. 45:103, 1989). We transformed the system into Birkhoff's normal form. Lie transforms reduce the system to an equivalent simpler system which is immediately solvable. Applying Arnold's theorem, we have found non-linear stability criteria. We conclude that L_6 is stable. We plotted graphs for (ω_1, D_2). They are rectangular hyperbola.

Keywords Non-linear stability · Photogravitational · Non-planar · Oblate primary · RTBP

1 Introduction

Hori (1966, 1967) applied a theorem by Lie in canonical transformation to the theory of general perturbations. Theorem is applicable to such cases where the undisturbed portion of Hamiltonian depends on angular variable as well as momentum variables. Deprit (1969) introduced the concept of Lie series to the cases where the generating function itself depends explicitly on the small parameter. Lie transforms define naturally a class of canonical mappings in the form of power series in the small parameter. They reviewed how a Lie series defines a canonical mapping as a formal power series of a small parameter ϵ, provided the generating function itself does not depend upon ϵ. This restriction is overcome by introducing Lie transform. They showed that how they naturally define the canonical transformations contemplated by Von Zeipel's method. Canonical mappings defined by Lie transforms as formal power series of a small parameter constitute the natural ingredient of Transformation Theory applied to Hamiltonian systems.

Orbital stability of quasi-periodic motions in multidimensional Hamiltonian systems was studied by Sokolskii (1978). With some applications to the Birkhoff's normal form along with its generalized form by K.R. Meyer, the restricted problem of three bodies near L_4, the Birkhoff's normalization procedure, and the singular perturbation, of Hamiltonian systems have been discussed by Liu (1985). Meyer and Schmidt (1986) established the full stability of Lagrange equilibrium point in the planar restricted three body problem even in the case when $\mu = \mu_c$. Hamiltonian is normalized up to order six and then KAM theory is applied. This establishes the stability of the equilibrium in degenerate case. Markeev (1966) and Alfriend (1970, 1971) have shown that L_4 is unstable when the mass ratio is equal to μ_2 or μ_3. The Lie transform method is an efficient perturbation scheme which explicitly generates the functional form of the reduced Hamiltonian under an implicitly defined canonical periodic near identity-transformation.

Coppola and Rand (1989) applied a method of Lie Transforms, a perturbation method for differential equations to a general class of Hamiltonian systems using computer algebra. They developed explicit formulas for transforming the system into Birkhoff normal form. They formed explicit nonlinear stability criteria solely in terms of H for systems where the linear stability is inconclusive. They applied these results to the non-linear stability of L_4 in the Circular RTBP. At L_4, Arnold's theorem (1961) must be used since a Lyapunov function cannot be found. They confirmed the previous computations of Deprit and Deprit-Bartholome (1967), Meyer and Schmidt (1986). Algorithms of linear and nonlin-

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