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ORIGINAL ARTICLE

On Sitnikov-like motions generating new kinds of 3D periodic orbits in the R3BP with prolate primaries

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Abstract The existence of new equilibrium points is established in the restricted three-body problem with equal prolate primaries. These are located on the Z-axis above and below the inner Eulerian equilibrium point L_1 and give rise to a new type of straight-line periodic oscillations, different from the well known Sitnikov motions. Using the stability properties of these oscillations, bifurcation points are found at which new types of families of 3D periodic orbits branch out of the Z-axis consisting of orbits located entirely above or below the orbital plane of the primaries. Several of the bifurcating families are continued numerically and typical member orbits are illustrated.

Keywords R3BP · Prolate primaries · Equilibrium points · Rectilinear motions · 3D periodic orbits

1 Introduction

The celebrated restricted problem of three bodies is well known to possess special solutions of rectilinear oscillatory motion along the Z-axis in the case of equal primaries. These are known in the literature as the Sitnikov motions (Sitnikov 1960; Perdios and Markellos 1988; Dvorak 1993; Belbruno et al. 1994; Lara and Buendía 2001). They originate at the inner collinear equilibrium point L_1, and are of special interest as generators of families of three-dimensional periodic orbits (Perdios 2007; Perdios et al. 2008). In the case where the primaries are oblate spheroids, Sitnikov motions also exist (Kalantonis et al. 2008).

It is also known that when oblateness of the primaries is taken into account the problem possesses equilibrium points in addition to the classical Eulerian and Lagrangian points. For example, such new points are known to be located in the X–Z plane almost directly above and below the oblate primaries for finite values of the mass parameter $\mu = m_2/(m_1 + m_2)$ (Douskos and Markellos 2006), while in the Hill case ($\mu \to 0$) with oblate secondary such equilibrium points are on the Z-axis above and below the secondary (Perdiou et al. 2005).

In the present paper we extend the above results by establishing the existence of new equilibrium points of the restricted problem with equal prolate primaries. These new equilibrium points are located on the Z-axis above and below the inner collinear equilibrium point L_1. They generate rectilinear periodic oscillations along segments of the Z-axis different to the Sitnikov motions, which in turn generate new families of three-dimensional periodic orbits. It is important here to note that these families could not have been discovered by classical methods (Robin and Markellos 1983).

The paper proceeds as follows. In Sects. 2 and 3, we describe the restricted problem in the case of equal primaries, in both mass and oblateness, establish the existence of the new equilibrium points, and determine their locations on the Z-axis. In Sect. 4, we describe the rectilinear motions arising from these equilibrium points. In Sect. 5, we consider the transversal stability of these motions under perturbations in the coplanar position and velocity components of the state

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