Free vibration analysis of non-uniform Euler–Bernoulli beams by Differential Quadrature Element Method (DQEM)

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Abstract

The differential equation of the free vibration of a non-uniform Euler–Bernoulli beam is considered. The stiffness and mass matrices of the structural system are produced by Differential Quadrature Element Method (DQEM). By solving the eigenvalue problem in determining the natural frequencies of the beam, extensive parametric studies based on the boundary conditions as well as the number of interpolation points are achieved. To verify the obtained results and also the rate of convergence of the employed method, the traditional Finite Element Method and the Galerkin Method are used. The results show the natural frequencies obtained by the DQEM have high accuracy while the CPU runtime decreases noticeably. Therefore, utilizing DQEM in the analysis of dynamic problems in non-uniform beams seems to be more efficient than the traditional Finite Element Method and the Galerkin Method as well.

Keywords: DQEM, free vibration, non-uniform beams, Finite Element Method, Galerkin Method.

1. INTRODUCTION :

Differential quadrature (DQ) was firstly introduced by Bellman and Casti [1] in 1971. In two recent decades, this numerical method attracted scientists and researchers more and more and its applications were extracted to solve more differential equations. The reasons for this attention are laid behind simplicity, accuracy and speed of calculation in solving differential equations with different boundary conditions.

In 1988, Bert and his co- workers first applied the DQ method to solve the structure mechanics [2, 3]. Since the DQ method has only the function values at grid points as the independent variables, difficulty arises for applying the boundary conditions, if the number of boundary conditions is greater than one. To overcome this problem, Bert and his co- workers introduced δ-point method. They could solve some of the problems by this method but all of the obtained results were not reliable because their accuracy was completely dependent on the problem and the boundary conditions.

Accordingly, DQ method was not flexible enough to solve structural problems. Striz et al. [4] first introduced the quadrature element method (QEM) to analyze beam structures with various loads, including discontinuous loads. However, the method is not quite convenient and accurate since the δ-point is introduced to establish the element equations. Later, Wang et al .[5] proposed a method to remove the δ-point in QEM by assigning two degrees of freedom to each end points for a fourth order differential equation. The method is called the differential QEM or simply, DQEM. All boundary conditions can be easily applied by DQEM and accurate solution can be obtained. The same idea is independently introduced by Chen et al. [6]. Recently Wu and his co-workers [7] proposed a generalized differential quadrature rule (GDQR). The GDQR constructs the DQ in a general situation and can easily deal with any problems involving more than one condition at any discrete point.