



## Axissymmetric circular shape opening in transversely isotropic half-space

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### Abstract

A half-space containing transversely isotropic material with a circular shape opening in it is considered in such a way the surfaces of the opening to be subjected to a time harmonic tensile tractions. The material axis of symmetry is considered to be perpendicular to the free surface of the half-space. The solution of this kind of problem may be used as a basic for fault analysis in transversely isotropic rocks. In order to analyze the problem, the Hankel integral transforms are used, so that the involved mixed boundary value problem is changed to a system of dual integral equations, which itself is transformed to Fredholm integral equations of second kind via the method propose by Noble in 1963. A numerical scheme is needed to solve the Fredholm integral equations in the general dynamic case. The graphical forms of the solution are presented to illustrate the effect of different degree of anisotropy on the response of the medium.

**Keywords:** Transversely isotropic; half-space; fault analysis; wave propagation; dual integral equations.

### 1. INTRODUCTION

The knowledge of the state of stress and displacement near cracks is one of the requirements for a fracture mechanics and thus the engineers are interested in. Because of advanced mathematical treatment needs to solve the involved boundary value problem, the mathematicians are also interested in the subject. There are many applications for the problem, and it is enough to say that the stress and the displacement fields from the solution of the 2-D elliptical crack problem are now the basis for fracture mechanics of brittle materials. Mathematically, the boundary value problem of this kind is similar to the problem of indentation of a medium with a rigid body, and the latter is an extension of Boussinesq problem. In the elastostatics of domains containing isotropic materials, extensive treatments have been given to the problems involving the stress distribution either under rigid discs or around cracks and notches affected by different load conditions, from which Sneddon [1; 2], Green [3] and Weaver [4] are mentioned as examples. Sneddon [1] showed that by applying Hankel transforms, the problem of distribution of stress in the neighborhood of a circular crack in a half-space is reduced to the problem of solving a pair of integral equations called dual integral equations [5; 6]. Green [3] showed that the crack problems can be reduced to a boundary problem in harmonic analysis which is analytically similar to the electrified disc problem. Weaver [4] applied boundary integral equation method, which is based on point load Green's functions and Betti-Rayleigh theorem, on an exterior problem to make some integral equations over the cracked area to find the dislocations. When a stress wave is disturbed by the presence of cracks, some of the waves are reflected and others are refracted, and the singular behavior in the elastodynamic stress field is found around the crack tip. These phenomena have received much attention in seismology, and thus the study of elastodynamic response with the events of scattering and singularity of existing cracks under dynamic loadings becomes increasingly important. Chen and Chang [7], by combination of the conventional finite element procedure and the Schwarz-Neumann technique, developed the so-called finite element alternating method, which is used for both two- and three-dimensional fracture problems with multiple cracks.