



# PID controller frequency-domain tuning for stable, integrating and unstable processes, including dead-time

Miroslav R. Mataušek\*, Tomislav B. Šekara

Faculty of Electrical Engineering, University of Belgrade, Serbia

## ARTICLE INFO

### Article history:

Received 25 May 2010

Received in revised form 23 August 2010

Accepted 3 September 2010

### Keywords:

PID controller tuning

PI controller tuning

Robustness

Measurement noise

## ABSTRACT

In the present paper a new tuning procedure is proposed for the ideal PID controller in series with the first-order noise filter. It is based on the recently proposed extension of the Ziegler–Nichols frequency-domain dynamics characterization of a process  $G_p(s)$ . Measured process characteristics are the ultimate frequency and ultimate gain, the angle of the tangent to the Nyquist curve of the process at the ultimate frequency, and  $G_p(0)$ . For a large class of processes the same tuning formulae can be effectively applied to obtain closed-loop responses with predictable properties. Load disturbance step responses without the undershoot and reference step responses with negligible overshoot are obtained by analyzing a test batch consisting of stable, integrating and unstable processes, including dead-time and oscillatory dynamics. The proposed tuning makes possible to specify the desired sensitivity to the high frequency measurement noise and the desired maximum sensitivity. Comparison with the optimal ideal PID controller in series with the first-order noise filter is presented and discussed. The extension of the proposed method to the PI controller tuning is direct. Comparison with the optimal PI controller is presented and discussed.

© 2010 Elsevier Ltd. All rights reserved.

## 1. Introduction

Design of the PID controller based on application of tuning formulae is initiated by Ziegler and Nichols [1] and still predominates over the optimization methods [2–5]. The basic idea of this approach is to find some transformations of a small amount of measured process characteristics into the values of the PID gains. For example, a refinement of the Ziegler–Nichols tuning formulae, proposed in [6], is based on estimation of the ultimate frequency  $\omega_u$ , ultimate gain  $k_u$  and the gain  $G_p(0)$  of a process  $G_p(s)$ . The same measured data are used in [7,8]. Time domain process dynamics characterization and PID controller tuning is initiated also by Ziegler and Nichols [1], used in [3,7,9] and developed further through the IMC-PID design and lambda-tuning in [10,11]. A detailed overview of tuning formulae developed until now is presented in [12]. Based on the frequency or time domain process dynamics characterization, they are developed to cover some specific process dynamic characteristics. Different tuning procedures are derived for stable, integrating and unstable processes.

In the present paper a unified PID controller tuning is proposed for a large class of processes (stable, integrating and unstable, including dead-time and oscillatory dynamics) satisfying condition

that the limit cycle exists, defined by  $k_u$  and  $\omega_u$ . Then, it is extended to the PI controller tuning.

According to [12] the greatest number of tuning formulae is derived for the ideal PID controller. The same holds true for the most of tuning formulae mentioned above. This means that the rules are derived for adjusting the proportional gain  $k$ , integral time  $T_i$  and derivative time  $T_d$ , taking the derivative (noise) filter time constant  $T_f$  equal to zero. Then,  $T_f$  can be determined as some fraction of the derivative time  $T_d$ , for example as  $T_f = T_d/N$ ,  $N = 2–10$ , resulting into deterioration of the performance/robustness tradeoff, obtained by the tuning rule proposed for  $N = \infty$ . Since in industry applications  $T_f$  must satisfy relation  $T_f > 0$ , the derivative (noise) filter must be an integral part of the PID optimization and tuning procedures [13]. In the present paper the ideal parallel PID controller in series with the noise filter  $F_{NF}(s) = 1/(T_f s + 1)$ , given by

$$C_{PID}(s) = \frac{k_d s^2 + k s + k_i}{s(T_f s + 1)} \quad (1)$$

is used, where the integral and derivative gains are related with the proportional gain  $k$  by the relation  $k_i = k/T_i$  and  $k_d = kT_d$ , respectively. Tuning method derived here, as well as optimization procedure used for comparison [5], include adjustment of the four parameters  $k$ ,  $k_i$ ,  $k_d$ ,  $T_f$ . When these parameters are determined, and controller  $C(s)$  in Fig. 1 is realized as PID controller (1), then  $G_{ff}(s)$  can be designed and tuned as in [2]. Also, when  $k$ ,  $k_i$ ,  $k_d$  and  $T_f$  are determined,  $C_{PID}(s)$  can be implemented as in (1) or in the traditional form, where noise filtering affect the derivative term only [4].

\* Corresponding author at: Department of Control, University of Belgrade, Bulevar Kralja Aleksandra 73, Belgrade 11000, Serbia. Tel.: +381 11 324 84 64; fax: +381 11 324 86 81.

E-mail addresses: [matausek@etf.rs](mailto:matausek@etf.rs) (M.R. Mataušek), [tomi@etf.rs](mailto:tomi@etf.rs) (T.B. Šekara).