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Frequency parameterization of H_∞ PID controllers via relay feedback: A graphical approach

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ABSTRACT

This paper focuses on a graphical approach to determine the region of proportional-integral-derivative (PID) controllers in the parameter space for which the closed-loop system is internally stable and the H_∞ optimization criteria are satisfied for a class of single-input single-output arbitrary order plant with or without dead time. Unlike conventional methods which the analytical models, such as transfer functions and state space models, are needed, the design information of the proposed approach is only the frequency response data, which are directly calculated from a single relay test for stable plants, or extracted from the closed-loop system frequency response data by dividing out the known stabilizing compensator for unstable plants using relay feedback methods. It is shown that the problem to be solved can be translated into simultaneous stabilization of the closed-loop characteristic function and a family of characteristic functions. Based on the technique of *D*-decomposition, the analytical boundaries of root invariant regions are derived and the admissible H_∞ region in the parameter space is the intersection of the admissible sets, and it can be drawn and identified immediately, not to be computed mathematically. A practical algorithm of determining the H_∞ region is proposed and two examples are used to illustrate the proposed method.

1. Introduction

In recent years, an interesting research in the direction of finding ways of computing all stabilizing regions in the parameter space of PID controllers has captured the attention of several researchers [1–10]. This fact is that the PID controller has just three tuning parameters, and the parameter space approach is suitable for the control design. Over the last 40 years control theory literature has been dominated by modern optimal control theory and its offshoots, such as the powerful H_∞ theory [11–13], but the resulting controllers may be of a very high order, sometimes even exceeding that of the original system. Since the H_∞ theory does not allow one to constrain the order of the designed controller, its direct application to the design of controllers of a given structure, such as PID controllers, faces substantial difficulties. Fortunately, there have been many researchers in attempting to combine the power of optimal control with low order/fixed structure controllers [14–18]. For example, based on the generalization of the Hermite–Biehler theorem, Ho [15] proposed a linear programming characterization of all admissible H_∞ PID controllers for a given plant. Keel and Bhattacharyya [19] developed a technique to compute the set of all first order stabilizing controllers which satisfy an H_∞ constraint for a given but arbitrary linear time-invariant plant based on determination of root invariant regions via *D*-decomposition and parameter mapping. An alternative approach to the design of low order controllers of a given structure which satisfy the H_∞-criterion was presented in [18]. But these methods rely on the fact that plant parameters are known, i.e., analytical models of plants, such as transfer function or state space models, must be constructed first.

However, in real world systems, there exist many situations where such exact information of modeling is unavailable or is difficult to obtain in practical applications. On the other hand, it is often the case that the time series data or frequency response of the plant can be easily measured experimentally. Recently, the approaches in [10,19,20] were developed to determine the entire set of stabilizing first-order/PID controller only based on the frequency response of a given LTI plant. Based on the concept of *D*-decomposition, the methods of determining the stabilizing region of PID-type controllers and first order controllers were proposed in [10] and [19], respectively. Using the sign change properties and the generalized Hermite–Biehler theorem under some reasonable assumptions, a method of computation of all stabilizing PID controllers was given in [20]. But the above results cannot be directly applied to the plants with dead time. Lately, in [21], it is shown that the entire sets of three term controllers achieving stability and various performance specifications can be found from the Nyquist–Bode data

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