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Statistical monitoring of nonlinear profiles by using piecewise linear approximation

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ABSTRACT

In many practical situations, the quality of a process, or product, is better characterized and summarized by the relationship between a response variable and one or more explanatory variables. Such a relationship between the response variable and explanatory variables is called a profile. Recently, profile monitoring has become a fertile research field in statistical process control (SPC). To handle the nonlinear profile data, the proposal considered in this paper is that the entire curve is broken into several segments of data points that exhibit a statistical fit to the linear model, and therefore each of them can be monitored separately by using existing linear profile SPC methods. A new method that determines the locations of change points based on the slop change is proposed. Two goodness-of-fit criteria are utilized for determining the best number of change points to avoid over-fitting. Two nonlinear profile examples taken from the literature are used to illustrate the proposed change-point model. Monitoring performances using the existing T^2 and EWMA-based approaches are presented when the nonlinear profile data is fitted by using the proposed change-point model.

1. Introduction and backgrounds

Statistical process control (SPC) has been successfully implemented in a variety of industries [1,2]. In most SPC applications, it is assumed that the quality of a process or a product can be adequately represented by the distribution of a univariate quality characteristic. However, in some particular situations, the quality of process or product is better characterized and summarized by a functional relationship between a response variable and one or more explanatory variables [3]. In such cases, the observed data points can be represented by some profiles, and then monitoring the profile is much more desirable than only monitoring their characteristic. Profile monitoring appears to be more common in practice since the pioneering work advocated by Kang and Albin [4]. Profile modeling/monitoring is, in essence, related to applied statistics and can be classified as linear and nonlinear profiles on the basis of functional relationship. Linear profiles can be well represented by a model like those used in linear regression analysis. By contrast, nonlinear profiles cannot be adequately represented by a linear model. From a different point of view, the monitoring of nonlinear profiles can be deemed as a particular application of multi-stage or multivariate process control task. Yet, very limited work has been done to address the monitoring of nonlinear profiles in the literature. Effective monitoring of nonlinear profiles is generally challenging in nature. One immediate difficulty would be how to characterize the nonlinear profile, followed by monitoring the original profile data or the associated parameters via transformation. The nonlinear profile is frequently regarded as high-dimensional data and analyzed by nonparametric methods. Some methods have been employed to reduce the dimensionality of multivariate data while preserving the data-clustering structure, such as wavelets, principle component analysis (PCA), independent component analysis (ICA), among others.

Process monitoring based on control charting usually consists of two phases. In Phase I, historical data is examined to investigate the sources of variation and to estimate in-control process parameters. In Phase II, control limits obtained from Phase I are used to monitor the on-going process. During each sampling time interval, the observed data points can be represented by a curve (*i.e.*, profile). The collection of data points is referred to as profile data. For each profile, it is assumed that several values of the response variable are measured along with the pre-specified levels of one or more explanatory variables. Therefore, the primary objective is to monitor the profile representing such a relationship instead of the average of a single quality characteristic. Kang

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