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# Min–Max MPC based on an upper bound of the worst case cost with guaranteed stability. Application to a pilot plant

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ABSTRACT

Min–Max MPC (MMMPC) offers the possibility to consider disturbances and uncertainties in the mathematical model used to predict the future trajectory of the system. The explicit consideration of disturbances and uncertainties in order to obtain a more robust control performance complicates the practical implementation of MMMPC due to the high computational burden required to compute the control law. The computational complexity of the optimization problem can be reduced by using approximate solutions or upper bounds of the worst case cost of the objective function. A computationally efficient MMMPC strategy based on such an upper bound was presented in a previous work also published in this journal (see Section 1). One of the main drawbacks of that strategy is the lack of a stability guarantee. In this paper it is shown that input-to-state practical stability of the MMMPC strategy can be guaranteed if a certain initial condition and a semi-feedback approach are used. Furthermore, the MMMPC strategy is validated in experiments with a continuous stirred tank reactor in which the temperature of the reactor is controlled. The behavior of the system and the controller are illustrated by means of experimental results.

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## 1. Introduction

Min–Max Model Predictive Control (MMMPC) offers the possibility to consider disturbances and model uncertainties in the prediction model within the framework of Model Predictive Control (MPC). In MMMPC strategies, the optimal control signal is computed minimizing the worst case cost. The mentioned worst case cost can be calculated maximizing the considered cost function with respect to all possible cases of disturbances and uncertainties [1,2]. The main drawback of this approach is the computational burden required to calculate the control signal by solving the resulting optimization problem. The optimization usually includes the solution of an NP-hard problem [3,4]. As a consequence of the numerical complexity to determine the control signal, the number of applications of MMMPC strategies is very small, even when there is evidence that they work better than standard MPC strategies in processes with uncertain dynamics or disturbances [5].

It is well known that the MMMPC control law based on linear models is piecewise affine when a 1-norm [6,7] or quadratic [8] criterion is used in the cost function. This property enables the possibility to build explicit forms of the control law with a reduced complexity [9]. Such explicit forms can be evaluated very fast provided that the complexity of the state space partition is moderate, which is the case for many applications. However, if the process model or the controller tuning parameters change, the computation of the controller has to be redone.

In Ref. [10], a computationally efficient MMMPC strategy in which the worst case cost is approximated by an upper bound has been presented. This strategy is based on a diagonalization algorithm which has a much lower computational burden than LMI techniques which have been proposed to obtain upper bounds [11,12]. The mentioned algorithm uses only simple matrix operations and can be implemented even with programming languages not destined for mathematical calculations, commonly found in industrial embedded systems. However, one of the main drawbacks of that strategy is the lack of a stability guarantee.

In this work, input-to-state practical stability is proven for the MMMPC strategy presented in Ref. [10]. Stability of the mentioned MMMPC strategy is guaranteed for a certain initial condition in the optimization procedure and under consideration of a semi-feedback approach [13]. Furthermore, the control strategy is validated in experiments with a continuous stirred tank reactor (CSTR). The used system emulates the heat produced by an exothermic chemical reaction using an electric resistance. It has been used before as a benchmark system for nonlinear and Min–Max Model Predictive Control strategies [14,15]. The nonlinear process dynamics are approximated by means of a linear model with additive and bounded uncertainties. The MMMPC strategy is implemented with the identified model and robustness of the control strategy can be

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