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Lyapunov stability of economically oriented NMPC for cyclic processes

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ABSTRACT

Several applications in process industries, such as simulated moving bed (SMB) separation and pressure swing adsorption (PSA), exhibit cyclic steady state behavior. Moreover, it is of economic interest to require energy intensive applications to take advantage of the periodically varying electricity price by changing the operating point frequently. Because traditional two-layer optimization methods are difficult to apply to these systems, we consider instead an economically oriented nonlinear model predictive control (NMPC) that directly considers system's economic performance subject to the dynamic model. On the other hand, the commonly used Lyapunov framework to analyze the stability for the economically oriented NMPC cannot be applied directly. This work proposes two economically oriented NMPC formulations and proves nominal stability for both. We introduce transformed systems by subtracting the optimal cyclic steady state from the original system, for which the Lyapunov function can easily be established. Moreover, we show that the asymptotical stability of the transformed system is equivalent to that of the original system. Hence, the original systems are also nominally stable at the cyclic optimal solution. Finally, an industrial size air separation unit case study with periodic electricity cost is presented.

1. Introduction

A number of on-line optimization tasks in the process industries are addressed by decomposing the plant's economic optimization into two layers. The upper level is normally real-time optimization (RTO), which optimizes the current economic objective using steady state first-principle models. The optimized steady state is passed onto the lower-level control layer as a set point. The advanced control layer minimizes the deviation between the plant and the set point to track it as fast as possible. Model predictive control (MPC) or nonlinear MPC (NMPC) is widely employed as the advanced controller, due to the advantages of handling constraints and multi-input–multi-output systems. However, it has been pointed out in [1,2] that this two-layer structured method is based on the assumption that model disturbance and transients are neglected in the RTO, which may not be appropriate for some applications. Moreover, the model inconsistency in the two layers and the presence of the transient constraint may lead to an unreachable set point [3]. In addition, since the NMPC layer does not see the economic performance, it may be suboptimal to minimize the transition time and simply track the set point as fast as possible [4].

Moreover, it is important to deal with systems that may not go to steady state. For instance, several applications in the process industry exhibit cyclic steady state behavior due to their operational nature, such as pressure swing adsorption (PSA) [5] and simulated moving bed (SMB) separation [6]. To deal with these periodic systems, Lee et al. [7] proposed a tracking-MPC method by using the concept of repetitive control, but without considering economic performance. Furthermore, periodically varying power prices suggest that it is difficult to achieve optimal economical performance by running the plant at a steady state; a periodic operation which takes advantage of the varying electricity price is preferred. For the above applications, the number of time steps in an operating period are known in advance and unaffected by optimization. Moreover, it is difficult to implement the traditional two-layer structured method since there is no optimal steady state.

As a result, interest has significantly increased in economically oriented NMPC which directly optimizes the plant's economic performance subject to dynamic constraints. Recently, many NMPC practitioners have reported good practical performance by using heuristic economically oriented NMPC formulations [8,4,1,9]. However, stability theory supporting economically oriented NMPC is still in early stages [10–12], in contrast to the mature theoretical basis of set point tracking NMPC, where one can find good reviews [13–15] of Lyapunov-based stability theory. This is partially due

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