



Robust adaptive neural observer design for a class of nonlinear parabolic PDE systems

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ABSTRACT

A robust adaptive neural observer design is proposed for a class of parabolic partial differential equation (PDE) systems with unknown nonlinearities and bounded disturbances. The modal decomposition technique is initially applied to the PDE system to formulate it as an infinite-dimensional singular perturbation model of ordinary differential equations (ODEs). By singular perturbations, an approximate nonlinear ODE system that captures the dominant (slow) dynamics of the PDE system is thus derived. A neural modal observer is subsequently constructed on the basis of the slow system for its state estimation. A linear matrix inequality (LMI) approach to the design of robust adaptive neural modal observers is developed such that the state estimation error of the slow system is uniformly ultimately bounded (UUB) with an ultimate bound. Furthermore, using the existing LMI optimization technique, a suboptimal robust adaptive neural modal observer can be obtained in the sense of minimizing an upper bound of the peak gains in the ultimate bound. In addition, using two-time-scale property of the singularly perturbed model, it is shown that the resulting state estimation error of the actual PDE system is UUB. Finally, the proposed method is applied to the estimation of temperature profile for a catalytic rod.

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1. Introduction

In most industrial processes, the total state vector can seldom be measured and the number of outputs is much less than the number of states. In addition, the process measurements are often corrupted by significant experimental error, and the process itself is subject to external disturbance. Without some consideration of these problems in the total control system design, the measurements used for feedback control will often be inadequate for acceptable control system performance. Thus, it is of theoretical and practical importance to study how to provide acceptable estimates of all the state variables in the face of measurement error and process disturbances, which is central to the control system design. Over the past decades, a large number of powerful results on state estimation are available for linear or nonlinear systems described by ordinary differential equations (ODEs), such as the well-known linear Kalman filter [1], linear H_∞ filter [2], nonlinear H_∞ filter [3,4], and H_∞ adaptive observer [5]. However, most practical industrial processes are inherently distributed in space and time, which are usually described by a set of nonlinear partial differential equations (PDEs).

Compared with ODE systems, there exist few results on state estimation for these PDE systems.

There are many industrially important diffusion–convection–reaction processes which are naturally described by nonlinear parabolic PDEs. Examples include rapid thermal processing, plasma reactors, crystal growth processes to name a few. The main feature of parabolic PDE systems is that the eigenspectrum of the spatial differential operator can be partitioned into a finite-dimensional slow one and an infinite-dimensional stable fast complement. Motivated by this fact, the standard approach to state estimation of linear or semilinear parabolic PDE systems is to obtain an approximate ODE representation of the original PDE system by utilizing the spatial discretization techniques [6,7], which is then used for estimator design purposes. In [6,8,9], the standard Galerkin method was used to derive a finite dimensional ODE model by simply neglecting the effect of the infinite-dimensional stable fast complement, which may require keeping a large number of modes to derive an ODE model that yields the desired degree of approximation, leading to high dimensionality of the resulting estimators. To overcome the problem of high dimensionality, the singular perturbation formulation (SPF) of Galerkin's method, proposed for the finite-dimensional state feedback control design of nonlinear parabolic PDE systems [10,11], was employed to design a nonlinear state observer for constructing a finite-dimensional output

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