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Damping feedback stabilization for cyclic interconnections systems: Oscillations suppression and synchronization

Nicolas Hudon [a], Martin Guay [b,*]

[a] School of Chemical Engineering, The University of New South Wales, UNSW Sydney, NSW 2052, Australia
[b] Department of Chemical Engineering, Queen's University, Kingston, ON K7L 3N6, Canada

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ABSTRACT

This paper considers the problem of stabilizing nonlinear systems in the form of cyclic interconnections. This class of systems is of interest in process control, as some (bio)chemical reactions can be described as cyclic interconnections. The objective of the paper is to present a constructive approach to compute a dissipative potential function for the system. This potential is then used for constructing a smooth feedback stabilizing controller. We first obtain a characteristic one-form for the system by taking the interior product of a non vanishing two-form with respect to the drift vector field. A homotopy operator is then constructed locally around the desired equilibrium, leading to the computation of a dissipative potential for the system. The dynamics of the system is then decomposed into an exact part and an anti-exact one. The exact part is generated by a potential, that is used to construct a smooth stabilizing feedback controller under the weak Jurdjevic–Quinn conditions. We consider the problems of oscillations suppression and synchronization of oscillators as illustrations of potential applications of the proposed construction. The potential of the method is illustrated through an example of a system given as a cyclic interconnection.

1. Introduction

This paper studies feedback stabilization of a particular class of chemical and biochemical reaction networks represented as cyclic interconnections. The problem of stability characterization for systems represented in this particular form has been study extensively in the literature. An example of systems with interconnections is metabolic network with feedback inhibition, studied for example in [10]. The reader is referred to the exposition in [2] for references on the stability characterization problem. The important contributions in [2] was to relate diagonal stability to a secant criterion in the context of interconnected system and to provide a constructive stability proof for cyclic systems. Extensions of these results to complex (bio)chemical reaction networks are given in [3]. In the general case, cyclic systems can be represented graphically as in Fig. 1, where each block can be of the form

$$H_i : \begin{cases} \dot{x}_i = -f_i(x_i) + g_i(x_i)u_i \\ y_i = h_i(x_i) \end{cases} . \quad (1)$$

This input/output representation was considered for example in [21] for the synchronization of coupled nonlinear Goodwin oscillators using incremental dissipativity.

In this paper, we look at the problem of constructing smooth feedback stabilizers for this class of systems, where a control $u(x)$ is assumed to be injected at any point in the structure. In recent years, energy-based methods were developed and studied extensively for nonlinear controller design. In particular, the representation of nonlinear systems as generalized Hamiltonian systems (see for example [19]) generated many successful controller design strategies for systems possessing an "energy-like" function or a storage function. However, for mass balance systems, *i.e.*, for reaction networks, such generating function of the dynamics might be difficult to develop from the physics of the system. The problem of dissipative Hamiltonian representation of a reaction system was given in [20]. In [18], feedback stabilization of a Lotka–Volterra system using a related passivity-based approach was considered. In that case, the authors solved a set of partial differential equations, known in the literature as the matching equations. In general, the problem of deriving an energy-type function, that can be used as a Lyapunov function, from a physical point of view, is a challenging task for mass and energy balance systems that appear in chemical process control. In recent years, following the contributions [1,11,12], potential functions derived from thermodynamics were used for the analysis of controlled chemical systems, including the analysis of CSTR using entropy in [8]. The approach presented in the present paper

* Corresponding author. Tel.: +1 613 533 2765; fax: +1 613 533 6637.
E-mail addresses: martin.guay@chee.queensu.ca, guaym@chee.queensu.ca (M. Guay).

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