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# Optimized fractional order conditional integrator

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#### 1. Introduction

The integrator is widely used to get the zero steady-state error in the control systems. However, the 90° phase lag at all frequencies, which means a loss in relative stability, is the cost for the benefit of zero steady-state error. The reset or Clegg conditional integrator (CCI) was proposed for the first time in Ref. [4] to reduce this phase lag while retaining the integrator's desirable magnitude frequency response. CCI provides more flexibilities in linear controllers design. The potential advantages of using CCI to meet some stringent design specifications have been presented in Refs. [1,2,8,11,25].

In Refs. [12,24], an intelligent conditional integrator (ICI) was presented. In ICI, the integrator "holds" the output value from the zero-crossing point of the input signal derivative to the zero-crossing point of the input signal, and resets the output value at the zero-crossing point of the input signal. The phase delay of ICI was reduced further compared with CCI. In order to achieve bet-

### ABSTRACT

In this paper, an optimized fractional order conditional integrator (OFOCI) is proposed. The key feature of this OFOCI is that, the fractional order of the integrator and the output parameter for the "hold" value in the course of conditional integration, can be tuned following the analytical optimality design specifications, to achieve the optimized performance not achievable for integer order conditional integrators (IOCIs). The proposed optimality specifications are given to satisfy the desired phase delay of the conditional integrator, and to minimize the concerned high order harmonic magnitude ratio. The numerical solution of calculating the optimized parameters of the OFOCI is introduced. The phase delays and the magnitude ratios of four OFOCIs designed according to four different phase delays are compared with the IOCIs. Simulation results with the FFT spectra are also presented to illustrate the theoretical analysis.

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ter control performance, we can proportionally tune a parameter k to change the "holding" output value of ICI in between the zerocrossing point of the input signal derivative and the zero-crossing point of the input signal. This output tunable conditional integrator can be treated as a modified intelligent conditional integrator (MICI).

On the other hand, from the very beginning of using the fractional calculus in control [18,20,23], the fractional integrator has been considered as an alternative approach for control purposes [7,10,13–15,17]. So, the fractional integrator can also be used in the conditional integrator as the fractional order conditional integrator (FOCI). The potential control performance improvement may be achievable with an extra tunable parameter—fractional order  $\alpha$  of FOCI. Meanwhile, the describing function is widely applied for the analysis of the nonlinear systems control [19,22,5,16]. Actually, the conditional integrator which is a nonlinear block can be analyzed using the describing function method.

In this paper, an optimized fractional order conditional integrator (OFOCI) is proposed. By tuning the fractional order  $\alpha$  and the parameter *k* for changing the output "holding" value following the analytical optimality design specifications, this presented OFOCI can achieve the optimized performance not achievable for integer order conditional integrators. The optimality specifications for FOCI are proposed to satisfy the desired phase delay in the feasible

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