



# Moving horizon estimation for distributed nonlinear systems with application to cascade river reaches<sup>☆</sup>

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## ABSTRACT

This paper presents a moving horizon estimation (MHE) method for discrete-time nonlinear systems decomposed into coupled subsystems with non-overlapping states. In the proposed algorithm, each subsystem solves a reduced-order MHE problem to estimate its own state based on the estimates computed by its neighbors. Conditions for the convergence of the estimates are investigated. The algorithm is applied to a model of three river reaches.

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## 1. Introduction

Many industrial processes and physical systems are composed by a large number of interconnected units, such as industrial plants [21] and power networks [22]. Their control poses many algorithmic and technological problems, due for instance to their large-scale nature and to limitations in information exchange. These problems motivate the intense research activity in the design of distributed control systems, in particular with model predictive control (MPC) [19]. However, most of the distributed control methods proposed so far are state-feedback, so that in order to guarantee a fully distributed control design, also distributed state estimation algorithms dealing with constraints are needed.

Early works in distributed estimation were aimed at reducing the computational complexity of centralized Kalman filters by parallelizing computations, see e.g. [8,15], under the assumption that each subsystem has full knowledge of the whole dynamics. Subsequently, [13] proposed a solution based on the use of reduced-order and decoupled models for each subsystem, while subsystems with overlapping states were considered in [10,20,21].

More recently, consensus-based distributed state estimators for sensor networks where each sensor measures just some of the system outputs and computes the estimate of the overall state have been studied, e.g. see [14]. Along the same lines and in order to cope with constraint on noise and state variables, in [4,5] distributed moving horizon estimators (MHE) for sensor networks have been proposed. Partition-based MHE algorithms (PMHE) for linear constrained systems decomposed into interconnected subsystems without overlapping states have recently been developed in [6]. In these algorithms, each subsystem estimates its own states based on information transmitted by its neighbors.

The aim of this paper is to extend the results of [6] to the case of nonlinear systems. The convergence properties of the method are guaranteed under suitable sufficient conditions.

The proposed partition-based MHE is applied to the problem of estimating the levels and flow rates in a model of three cascade river reaches. Interconnections between successive reaches are due to the dependence of the input flow rate of the downstream reaches to the level of the final section of the upstream ones, which cannot be measured, but just estimated from the available measures collected along the reach.

The paper is structured as follows. Section 2 introduces nonlinear partitioned systems and the main assumptions concerning their dynamics. Section 3 describes the proposed MHE algorithm, while convergence results are provided in Section 4. The illustrative example is considered in Section 5. For the sake of readability, the proofs of the main results are collected in an Appendix A.

*Notation:*  $I_n$  and  $0$  denote the  $n \times n$  identity matrix and the matrix of zero elements whose dimensions will be clear from the

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