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Shaping probability density functions using a switching linear controller

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ABSTRACT

Linear controllers for dynamic systems have been used in a stochastic context to decrease the variance of certain key variables. An extension is to use nonlinear controllers for shaping the entire probability density function (PDF). Typically, polynomial controllers are preferred due to their flexibility and also because their continuity makes them mathematically attractive. However, due to restrictions on stability, these controllers tend to be quasi-linear around the desired operating point. So, in this paper, a switching linear controller is presented as an alternative for PDF shaping. It is shown that, despite the discontinuity, an analytical solution of the PDF can be obtained using the Fokker–Planck–Kolmogorov equation. Also, for a simple case, even the Gram-Charlier coefficients can be computed analytically. Thus, it is demonstrated that the PDF shaping problem with a switching controller does not cause any further complications than using polynomial controllers. It is then shown in simulation that switching controllers are more robust and could create more skewness due to their discontinuity.

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1. Introduction

One of the goals of process control is to attenuate the effect of various perturbations occurring in an industrial process. The most common approach is to reduce the variance by using an appropriate linear controller. If the objective function is symmetrical, a symmetrical reduction in the variance corresponds to an increase in the expected value of the cost function. However, in many optimization problems, the objective function is asymmetrical, especially when constraints are present. In a constrained optimization problem, violation of an active constraint would cause more degradation of the objective function (penalty) than the same amount of variation on the safe side. For such problems, the typical approach is still to reduce the variance and shift the set point closer to the constraint for economic gain [1–3].

However, recent studies have concentrated on the fact that the economic gain can be further improved by shaping the output probability density function (PDF), so as to take advantage of the asymmetrical features present in the economic cost function [4–6]. In this context, two classes of work have been reported in the literature, (i) directly maximizing the economic gain with a nonlinear controller, i.e., the shape of the PDF being implicit and (ii) choosing an asymmetric target PDF and adjusting the controller to match the chosen PDF closely. The work reported by Speyer et al. [7,8], falls under the first category, where a non-quadratic cost function is used. A predetermined PDF was not imposed, but the resulting controller not only reduces the variance but changes the shape of the entire output PDF. In the second category, the emphasis is more on shaping the PDF rather than optimizing a non-quadratic cost function. All the methods in this category have the same structure, i.e., (i) introducing a non-linearity, parameterized in a certain manner, (ii) computing an approximate or parameterized version of the PDF and (iii) choosing an appropriate measure to characterize the distance between the desired and output PDFs.

Most of the methods available in the literature use simple polynomial controllers to induce nonlinearity. The only exception is the method by Wang [9] that use B-splines. The main advantage of having simple polynomial controllers is its continuity, which is a mathematically attractive property. However, to induce strong asymmetry requires a lot of terms and such controllers are less robust and can become unstable for operating points far from the mean.

Secondly, the exact solution of the PDF is difficult to compute when polynomial controllers are used. So, one of the approaches parameterizes the PDF using Gram-Charlier basis functions and derives analytical expressions for the Gram-Charlier coefficients [10–12]. An alternative is to work with the probability potential, a concept of probability potential is motivated by Fokker–Planck–Kolmogorov (FPK) equation [13]. Another proposal deals with the approximation of the PDF using a neural-network [14–21].

The last aspect is the choice of a measure for the distance between PDFs. It depends closely on how the PDF is computed. When Gram-Charlier coefficients are used to describe the PDFs, the sum of the squared differences between the coefficients is considered as a measure of the distance between the PDFs [10–12].

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