Journal of Process Control 21 (2011) 1459–1466

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Partial enumeration MPC: Robust stability results and application to an unstable CSTR☆,☆☆

Gabriele Pannocchia [a,*], Stephen J. Wright [b], James B. Rawlings [c]

[a] Dept. of Chemical Engineering (DICCISM), Univ. of Pisa, Pisa, Italy
[b] Computer Science Dept., Univ. of Wisconsin, Madison, WI, USA
[c] Chemical and Biological Engineering Dept., Univ. of Wisconsin, Madison, WI, USA

ARTICLE INFO

Article history:
Received 26 November 2010
Received in revised form 17 June 2011
Accepted 17 June 2011
Available online 31 July 2011

Keywords:
Partial enumeration MPC
Explicit control laws
Inherent robust stability
Lyapunov functions

ABSTRACT

We describe a partial enumeration (PE) method for fast computation of a suboptimal solution to linear MPC problems [1] with robust stability properties. Given that the suboptimal PE-based control law is non-unique (that is, a set-valued map) and (possibly) discontinuous, we treat the closed-loop system, appropriately augmented, as a difference inclusion. We derive novel robust exponential stability results for difference inclusions of this type. In particular we show that *Strong Robust Exponential Stability* (SRES) holds, for *any* sufficiently small but otherwise arbitrary perturbation. Such approach allows us to show SRES of the closed-loop system under PE-based MPC. Application to a simulated open-loop unstable CSTR with separation unit and recycle is presented to show performance and timing results for PE-based MPC, as well as to highlight its robustness to process/model mismatch, disturbances and measurement noise.

1. Introduction

During the past decade, significant research activity in Model Predictive Control (MPC) has been devoted to the implementation of efficient methods for solving the associated constrained optimal control problem, which in the case of linear systems subject to linear constraints and quadratic performance function can be cast as a quadratic program (QP). Several methods rely *exclusively* on efficient on-line calculations [2–5], whereas explicit control law methods [6–9] move the most expensive calculations offline, limiting the online computations to a table lookup involving simple matrix/vector multiplications and inequality checks. Because of the exponential explosion of the number of table entries required as a function of the problem size (number of inputs, states, and constraints), explicit methods are limited to small systems. A method that can be considered in the middle ground between explicit methods and online optimization is partial enumeration [1,10], in which a table (with entries equivalent to those of explicit methods) of fixed size is scanned online to find the optimal control input. If none of the entries is optimal, a quick suboptimal input is computed, and the table is updated to include the new optimal entry for future decision times. (The least-recently used optimal entry is discarded if necessary to make room in the table). In this way, as time goes on, the table adapts to new operating conditions and contains only the entries that are currently most likely to be optimal. There have been other proposals to combine online optimization with explicit methods; see for example [11,12].

The objective of this paper is to revise the partial enumeration (PE) approach and to make appropriate modifications with the goal of showing its inherent robust stability. To this end, we derive novel results for inherent robust stability of difference inclusions of the type that arise with suboptimal MPC, in general, and partial enumeration MPC, in particular. The rest of this paper is organized as follows. In Section 2 we revise the PE-MPC method. In Section 3, we derive from scratch novel robust stability results and apply such results to PE-MPC. A simulated application to the control of an unstable CSTR with separation and recycle is presented in Section 5, conclusions are drawn in Section 6, and additional derivations and proofs are reported in Appendix A.

Notation. Given a vector $x \in \mathbb{R}^n$, $|x|$ denotes the 2-norm; given a sequence of vectors $\mathbf{x}_k := \{x(j)\}_{j=0}^{k-1}$, we define $\|\mathbf{x}_k\| := \sup_{j=0,1,\ldots,k-1} |x(j)|$. The superscript$'$ indicates the transpose operator, $\mathbb{B}$ denotes the closed unit ball, $\mathbb{B} := \{x \in \mathbb{R}^n,\ |x| \leq 1\}$, $\mathbb{I}_{\geq 0}$ indicates the set of nonnegative integers; and the symbols I and 0 indicate the identity and zero matrices of appropriate dimensions.

☆ This work was supported by National Science Foundation (Grant CTS-0456694) and TWCCC members.
☆☆ A preliminary version of this paper was presented as a Keynote Lecture at the IFAC conference DYCOPS 2010 in Lueven (Belgium).
* Corresponding author. Tel.: +39 050 2217838; fax: +39 050 2217866.
E-mail addresses: g.pannocchia@diccism.unipi.it (G. Pannocchia), swright@cs.wisc.edu (S.J. Wright), rawlings@engr.wisc.edu (J.B. Rawlings).

0959-1524/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jprocont.2011.06.010