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On stress-state dependent plasticity modeling: Significance of the hydrostatic stress, the third invariant of stress deviator and the non-associated flow rule

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ABSTRACT

It has been shown that the plastic response of many materials, including some metallic alloys, depends on the stress state. In this paper, we describe a plasticity model for isotropic materials, which is a function of the hydrostatic stress as well as the second and third invariants of the stress deviator, and present its finite element implementation, including integration of the constitutive equations using the backward Euler method and formulation of the consistent tangent moduli. Special attention is paid for the adoption of the non-associated flow rule. As an application, this model is calibrated and verified for a 5083 aluminum alloy. Furthermore, the Gurson–Tvergaard–Needleman porous plasticity model, which is widely used to simulate the void growth process of ductile fracture, is extended to include the effects of hydrostatic stress and the third invariant of stress deviator on the matrix material.

1. Introduction

Plasticity describes the deformation of a material undergoing non-reversible changes of shape in response to applied forces. Our ancestors in ancient times already recognized the plastic behavior of metals as they attempted to make various tools and weapons. However, the scientific study of plasticity may justly be regarded as beginning in 1864 when Tresca published his results on punching and extrusion experiments and formulated his famous yield criterion (Tresca, 1864). This yield criterion was then used by Saint-Venant (1870) and Levy (1870) in their development of a theory of rigid-perfectly plastic solid. Another well known yield criterion was proposed by von Mises (1913) on the basis of purely mathematical considerations and later was interpreted by Hencky (1924) as plastic yielding occurs when the elastic shear-strain energy reaches a critical value. Von Mises also independently proposed equations similar to Levy's for rigid-perfectly plastic materials. Other important contributions in the early development of the plasticity theory include the works by Prandtl (1925), Reuss (1930), among others. Subsequently, within the scope of elastic–plastic materials under small deformation, the notation of yield in the stress space formulation was generalized to cover work-hardening materials and a unified theory of plasticity began to emerge after World War II (Hill, 1950; Mendelson, 1968). The flow theory is the most widely known theory of plasticity, which consists of a yield criterion, a flow rule, a hardening law and the loading–unloading conditions. The yield criterion determines the stress-state when yielding occurs, the flow rule describes the increment of plastic strain after yielding,

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