



# Shakedown and limit analysis of 90° pipe bends under internal pressure, cyclic in-plane bending and cyclic thermal loading

Haofeng Chen\*, James Ure, Tianbai Li, Weihang Chen, Donald Mackenzie

Department of Mechanical Engineering, University of Strathclyde, Glasgow G1 1XJ, United Kingdom

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## ABSTRACT

The Linear Matching Method is used to create the shakedown limit and limit load interaction curves of 90° pipe bends for a range of bend factors. Two load cases are considered i) internal pressure and in-plane bending (which includes opening, closing and reversed bending) and ii) internal pressure and a cyclic through wall temperature difference giving rise to thermal stresses. The effects of the ratios of bend radius to pipe mean radius ( $R/r$ ) and mean radius to wall thickness ( $r/t$ ) on the limit load and shakedown behaviour are presented.

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## 1. Introduction

90° pipe bends are common in piping systems, and it is vital to ensure that these components are designed against plastic collapse. Additionally, design against incremental plastic collapse (ratchetting) or low cycle fatigue failure (resulting from reversed plasticity, also known as alternating plasticity) must also be performed. The conservatism resulting from limiting the structure to purely elastic behaviour at stresses which never exceed the yield stress is often not acceptable and components are designed to exhibit elastic shakedown (which allows plastic deformation that does not result in either alternating plasticity or ratchetting).

The shakedown of structures has been studied by many researchers. The complexity of the phenomenon means that analytical solutions exist for only the simplest of geometries and loading. Incremental Finite Element Analysis (FEA) and Direct Cyclic Analysis in Abaqus [1] can be used to predict if elastic shakedown, alternating plasticity or ratchetting occurs but does not evaluate limits or boundaries between these different responses. Creation of the Bree-like [2] shakedown boundary diagrams requires many FEA calculations to be performed, which becomes very computationally expensive with complex geometries. Consequently, many direct methods have been created based upon the Koiter [3] kinematic and

Melan [4] static theorems to calculate the shakedown limit. Typical of these are the GLOSS R-node method [5], the Elastic Compensation Method [6], mathematical programming methods [7], the Elastic Modulus Adjustment Procedure [8] and the Linear Matching Method (LMM) [9,10]. The LMM has been shown to give accurate results for many geometries with complex load histories and temperature dependent material properties. LMM ABAQUS subroutines have been consolidated by the British Energy Generation Ltd R5 research program and are used in the assessment of plant components [11].

The LMM is used in this work to analyse the elastic shakedown behaviour of 90° pipe bends. First a summary of current literature regarding the shakedown of this geometry is given, followed by an explanation of the LMM procedure, the Finite Element model used and finally the results, discussion and conclusions.

## 2. Summary of limit and shakedown analysis of pipe bends

Fig. 1 shows a typical 90° pipe bend. Such bends are commonly described in terms of two ratios:  $r/t$  and  $R/r$ , where  $r$  is the mean pipe radius,  $R$  is the bend radius and  $t$  is the wall thickness. When these ratios are combined, they give the bend characteristic of the pipe,  $h$ :

$$h = \frac{R/r}{r/t} = \frac{Rt}{r^2} \quad (1)$$

\* Corresponding author. Tel.: +44 1415482036; fax: +44 1415525105.  
E-mail address: [haofeng.chen@strath.ac.uk](mailto:haofeng.chen@strath.ac.uk) (H. Chen).