



## Stress intensity factor equations for mixed-mode surface and corner cracks in finite-thickness plates subjected to tension loads

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### ABSTRACT

Normalized mixed-mode stress intensity factor equations are presented for deflected and inclined circular surface and corner cracks in finite-thickness plates under uniform remote tensile loading. The equations are obtained by performing non-linear regression analyses on the data from previous numerical solutions based on three-dimensional enriched finite elements. In the equations, the effects of deflection/inclination angles and plate thickness on mixed-mode stress intensity factors are included. The comparisons of normalized stress intensity factors from the equations with those of the finite element analyses show good agreement. Thus, it is concluded that, as a reasonable approximation, the presented equations can be used to assess stress intensity factors and fracture conditions of mixed-mode circular surface and corner cracks in finite-thickness plates.

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### 1. Introduction

Surface and corner cracks are encountered in practical applications quite often. In fact, most cracks initiate as surface or corner cracks, and then may become a through-thickness crack depending on loading and geometry of the cracked region. In the last three decades, increasingly higher consideration has been given to the surface crack problem and different methods have been developed for its solution. The finite element method has become the most widely used technique for solution of three-dimensional fracture problems and most efforts in this area so far, especially in the earlier stages of research and development, concentrated on mode-I load and boundary conditions. For example, the finite element solutions and their related empirical equations developed by Raju and Newman for mode-I surface and corner cracks [1,2] are still very widely used in many practical applications by the scientific and practicing engineering communities.

Although some surface and corner cracks are subjected to uniaxial or mode-I type loading, in many cases general type of loading exists or the crack plane is not perpendicular to a uniaxial stress direction. In this case, mixed-mode fracture conditions are encountered near the crack tip, and realistic assessment of fracture

conditions requires determination of mixed-mode stress intensity factors (SIF) accurately. Although plates form a small class of engineering structures that are dealt with in practice, three-dimensional fracture solutions for these types of geometries can generally be very useful in assessing the damage tolerance of some structures with general three-dimensional shape. In previous studies [3,4], mixed-mode stress intensity factor solutions were obtained using three-dimensional enriched finite elements and presented in the form of SIF plots along the crack front for different plate thicknesses and crack deflection/inclination angles. Although these graphical solutions can readily be used for fracture assessment of mixed-mode surface and corner cracks, it is not practical always to use graphs and it doesn't produce repeatable results every time the graph is read by the same person or when different people read it. Therefore, having equations that adequately represent the mixed-mode stress intensity factor distribution along the crack front would be very useful to consistently compute always same results and, more importantly, to be able to predict with higher accuracy the SIFs for intermediate plate thicknesses or crack deflection/inclination angles, for which there are no finite element solutions available.

Having been motivated by the needs stated above, in this study, mixed-mode stress intensity factor equations are presented for deflected and inclined semi-circular surface/corner cracks in plates under uniform tension loads. These equations are obtained by performing non-linear regression analyses [5] on previously generated mixed-mode stress intensity factors based on three-

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