International Journal of Pressure Vessels and Piping 88 (2011) 75–87

Contents lists available at ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

Exact and numerical elastodynamic solutions for thick-walled functionally graded cylinders subjected to pressure shocks

M. Shariyat*, M. Nikkhah, R. Kazemi

Faculty of Mechanical Engineering, K.N. Toosi University of Technology, Pardis Street, Molla-Sadra Avenue, Vanak Square, P.O. Box: 19395-1999, Tehran 19991 43344, Iran

ARTICLE INFO

Article history:
Received 5 January 2010
Received in revised form
21 January 2011
Accepted 21 January 2011

Keywords:
Analytical
Transfinite element
Functionally graded material
Elastodynamic
Dynamic loads
Thick-walled cylinder

ABSTRACT

In the present paper, analytical and numerical elastodynamic solutions are developed for long thick-walled functionally graded cylinders subjected to arbitrary dynamic and shock pressures. Both transient dynamic response and elastic wave propagation characteristics are studied in these non-homogeneous structures. Variations of the material properties across the thickness are described according to both polynomial and power law functions. A numerically consistent transfinite element formulation is presented for both functions whereas the exact solution is presented for the power law function. The FGM cylinder is not divided into isotropic sub-cylinders. An approach associated with dividing the dynamic radial displacement expression into quasi-static and dynamic parts and expansion of the transient wave functions in terms of a series of the eigenfunctions is employed to propose the exact solution. Results are obtained for various exponents of the functions of the material properties distributions, various radius ratios, and various dynamic and shock loads.

1. Introduction

Dynamic and shock loadings are among the common loadings for some engineering structures. Elastodynamic analysis of the structures may be used for extracting the transient dynamic response characteristics or determining circumstances where the dynamic strength is much higher than the static one [1–4]. Furthermore, the wave nature of the stress propagation may cause stress gradients that may lead to crack nucleation or propagation phenomena. In these circumstances, improper distribution of the material properties may exaggerate the problem and subsequently cause premature failures. Therefore, elastodynamic analysis is a vital stage in the design process of components subjected to rapid load variations or shocks. Among the non-homogeneous composite materials, functionally graded materials (FGMs) are used extensively due to smooth variations of the material properties and the capability to monitor variations of the volume fraction of the reinforcement phases within the microstructure. Thick cylinders fabricated from functionally graded materials which are especially mixtures of ceramics and metals are commonly used e.g. in aerospace, nuclear, automobile, and other industries.

Many of the well-known stress analyses performed so far for thick FGM cylinders are mainly performed for steady-state or transient conditions under purely thermal loads [5–9]. Some of these researches have been accomplished based on the multilayer discretization approximation [10–12].

Some researchers have performed elastodynamic and stress wave propagation analyses in thick-walled cylinders made of isotropic or special anisotropic cylinders under dynamic pressures. Santosa [13] used the direct and inverse Fourier transform for the displacements in an eigenfunction expansion to determine approximate transient responses of infinite isotropic cylinders. An exact elastodynamic solution is developed by Wang and Gong [14] to determine the dynamic stresses in a multilayered cylinder subjected to dynamic pressure employing the finite Hankel and Laplace transforms. Heyliger and Jilania [15] adopted a variational method to study the frequency response of inhomogeneous cylinders and spheres. Zhuang [16] presented a steady-state Green's function solution for a laminated anisotropic cylinder with uniform thickness layers. Ding et al. [17] expressed the displacement function in terms of multiplication series of Bessel and time functions to study transient responses of axisymmetric isotropic cylindrical shells subjected to dynamic loads. Chakraborty and Gopalakrishnan [18] studied wave propagation in anisotropic inhomogeneous layered media whose material properties vary according to an exponential function, using a spectral layer element. El-Raheb [19] studied effects of the circumferential and radial inhomogeneities on the transient waves of a hollow cylinder. The cylinder is divided into isotropic sub-cylinders. The static–dynamic superposition method is employed to determine the transient response. Later, he extended this work to

* Corresponding author. Tel.: +98 9122727199; fax: +98 21 88674748.
E-mail addresses: m_shariyat@yahoo.com, shariyat@kntu.ac.ir (M. Shariyat).

0308-0161/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijpvp.2011.01.005