A stochastic-spectral finite element method applied to the analysis of stochastic structural mechanics problems

Pooya Zakian[1], Naser Khaji[2]

1. Ph.D. Student in Earthquake Engineering, Faculty of Civil and Environmental Engineering, Tarbiat Modares University, P.O. Box 14115–397, Tehran, Iran
Corresponding Author's E-mail address: p.zakian@modares.ac.ir

2. Professor in Earthquake Engineering, Faculty of Civil and Environmental Engineering, Tarbiat Modares University, P.O. Box 14115–397, Tehran, Iran
E-mail address: nkhaji@modares.ac.ir

Abstract

In this article, stochastic finite element and spectral finite element methods are combined as a new hybridized numerical method for uncertainty quantification. A popular version of stochastic finite element method is used for the combination. In the other hand, spectral finite element method is a numerical method employing special orthogonal polynomials (e.g., Lobatto) and quadrature schemes (e.g., Gauss-Lobatto-Legendre), leading to desirable accuracy, and much less domain discretization with excellent convergence. The proposed method of this study is a hybrid method utilizing efficiencies of both methods for analysis of stochastically linear elastostatic problems. Additionally, spectral finite element is utilized for numerical solution of Fredholm integral equation followed by the proposed numerical method, one which increases the efficiencies. Numerical examples demonstrate abilities of the proposed method.

Keywords: Karhunen–Loève expansion, Polynomial chaos expansion, Stochastic finite element method, Spectral finite element method, Stochastic structural analysis

1. INTRODUCTION

Application of stochastic finite element method (StFEM), as an efficient numerical tool, to the analysis of realistic complex engineering problems is of great importance. For instance, the StFEM can quantify inherent uncertainly of an engineering system such as material properties, geometrical properties and so on. This method can also be used for incorporating random loadings such as earthquake, wind, wave loads, and so forth. The StFEM is an extended version of deterministic finite element method (FEM) for considering the random fluctuation of structural properties, loads, and responses. There are many publications dealing with the StFEM, among which some remarkable works are reviewed in this section. Deb *et al.*[1] solved stochastic partial differential equations utilizing Galerkin method. Kaminski [2, 3] carried out stochastic finite element analysis of elastostatic problems with perturbation based approach. Anders and Hori [4] developed the StFEM for analysis of elasto-plastic structures. Furthermore, recent achievements for improvement of computational efficiency of the StFEM may be found in references [5-8].

Spectral finite element method (SFEM) is a numerical method originally proposed for wave propagation problems for the sake of its high accuracy and excellent convergence properties. The SFEM is fundamentally a combination of two different methods of spectral method and the FEM. The SFEM was firstly presented by Patera [9] in computational fluid dynamics framework and then was applied to the various wave propagation problems. Solution of elastostatic and elastodynamic problems using Chebyshev SFEM was performed by Dauksher and Emery [10]. Khaji *et al.* [11] investigated time domain SFEM analysis of transient elastodynamic problems. Kudela *et al.* [12] presented the application of spectral finite elements to one-dimensional (1D) elastic wave propagation problems and verified numerical results experimentally. Witkowski *et al.* [13] carried out static and dynamic analysis of two-dimensional (2D) elastostatic and wave propagation problems employing several examples in order to demonstrate capabilities of the SFEM.