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Analytical yield criterion for an anisotropic material containing spherical voids and exhibiting tension–compression asymmetry

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ABSTRACT

A significant difference between the behavior in tension versus compression is obtained at the polycrystal level if either twinning or non-Schmid effects are contributors to the plastic deformation at the single crystal level. Examples of materials that exhibit tension–compression asymmetry include hexagonal close-packed (HCP) polycrystals and intermetallics (e.g., molybdenum compounds). Despite recent progress in modeling their yield behavior in the absence of voids, the description of coupling between plasticity and damage by void growth in these materials remains a challenge.

This paper is devoted to the development of a macroscopic anisotropic yield criterion for a porous material when the matrix material is incompressible, anisotropic and displays tension–compression asymmetry. The analytical yield criterion is obtained based on micromechanical considerations and non-linear homogenization. The matrix plastic behavior is described by the Cazacu et al. (2006) anisotropic yield criterion that is pressure-insensitive and accounts for strength–differential effects. Comparison between finite element cell calculations and theory show the predictive capabilities of the developed anisotropic model in terms of modeling the combined effects of anisotropy, tension–compression asymmetry of the matrix and voids on the overall yielding of the porous aggregate. It is shown that if the matrix material does not display tension–compression asymmetry, the developed criterion reduces to that of Benzerga and Besson (2001). If the matrix is isotropic, it reduces to the isotropic criterion developed in Cazacu and Stewart (2009).

1. Introduction

Ductile failure in metals occurs due to the nucleation, growth and coalescence of voids (McClintock, 1968; Rousselier, 1987). Voids are nucleated in metals mainly by decohesion at the particle–matrix interfaces or by micro-cracking of second-phase particles (see, for example, Tvergaard, 1981). Additionally, voids can nucleate in single crystals that contain neither pre-existing voids nor inclusions (see, for example, Cuitiño and Ortiz, 1996; Lubarda et al., 2004 and the recent studies on cylindrical void growth in rigid-ideally plastic single crystals of Kysar et al., 2005, 2006; Kysar and Gan, 2007). Thus, the ability to accurately describe the evolution of voids in a ductile metal is crucial to being able to accurately predict its failure.

Gurson (1977) developed widely used macroscopic yield criteria for porous metals containing either spherical or cylindrical voids and with the matrix obeying the von Mises isotropic yield condition. The success of Gurson's (1977) criterion lies in the fact that it was deduced based on micromechanical considerations. Several modifications of Gurson's (1977) criterion were proposed (see Tvergaard, 1981; Tvergaard and Needleman, 1984; Koplik and Needleman, 1988) based on finite element calculations to account for void interaction and coalescence. Gologanu et al. (1993, 1994, 1997) generalized Gurson's (1977) analysis by considering a spheroidal volume containing a confocal spheroidal cavity. In Garajeu (1995) and Garajeu et al. (2000) the overall response of a porous metal with a viscous matrix was investigated. Gurson's analysis has also been extended to the case when the matrix material is compressible and obeys a Drucker-Prager yield criterion (see Jeong and Pan, 1995; Guo et al., 2008).

Most metallic alloys display plastic anisotropy as a result of forming processes. Recent studies have been devoted to the experimental characterization of the anisotropy of fracture in different alloys (see, for example, Benzerga et al., 2004a, for an overview of experimental evidence for anisotropic ductile fracture in steels). Liao et al. (1997) extended Gurson's (1977) cylindrical criterion to account for transverse isotropy by using Hill's (1948) yield criterion for the matrix material; however, these authors assumed that the anisotropy in the plane of the sheet is weak and can be described by a single anisotropy coefficient. Benzerga and Besson

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