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Inhomogeneity of the first and second statistical moments of stresses inside the heterogeneities of random structure matrix composites

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ABSTRACT

In this paper linearly thermoelastic composite media are treated, which consist of a homogeneous matrix containing a statistically homogeneous random set of heterogeneities. Effective properties (such as compliance, thermal expansion, stored energy) as well as the first statistical moments of stresses in the phases are estimated for the general case of nonhomogeneity of the thermoelastic inclusion properties. The micromechanical approach is based on the generalization of the "multiparticle effective field" method (MEFM, see for references Buryachenko, Appl. Mech. Rev. (2001), 54, 1–47), previously proposed for the estimation of stress field averages in the phases. The method exploits as a background the new general integral equation proposed by the author before and makes it possible to abandon the use of the central concept of classical micromechanics such as effective field hypothesis as well as their satellite hypothesis of "ellipsoidal symmetry". The implicit recursion representations of the effective thermoelastic properties and stress concentration factor are expressed through some building blocks described by numerical solutions for both the one and two inclusions inside the infinite medium subjected to the inhomogeneous effective fields evaluated from subsequent self-consistent estimations. One also estimates the inhomogeneous statistical moments of local stress fields which are extremely useful for understanding the evolution of nonlinear phenomena such as plasticity, creep, and damage. Just at some additional assumptions (such as an effective field hypothesis) the involved tensors can be expressed through the Green function, Eshelby tensor and external Eshelby tensor. These estimated inhomogeneities of effective fields lead to the detection of fundamentally new effects for the local stresses inside the heterogeneities.

1. Introduction

The prediction of the behavior of composite materials in terms of the mechanical properties of constituents and their microstructure is a central problem of micromechanics, which is evidently reduced to the estimation of stress fields in the constituents. Appropriate, but by no means exhaustive, references for the estimation of effective elastic moduli of statistically homogeneous media are provided by the reviews Willis (1981), Mura (1987), Nemat-Nasser and Hori (1993), Torquato (2002), and Milton (2003), Buryachenko (2007a). It appears today that variants of the effective medium method (Kröner, 1958; Hill, 1965) and the mean field method (Mori and Tanaka, 1973; Benveniste, 1987) are the most popular and widely used methods. The multiparticle effective field method (MEFM) was also put forward and developed (see for references Buryachenko, 2001, 2007a). The MEFM is based on the theory of functions of random variables and Green's functions. Within this method one constructs a hierarchy of statistical moment equations for conditional averages of the stresses in the inclusions. The hierarchy is then cut by introducing the notion of an effective field. This way the interaction of different inclusions is taken into account. Thus, the MEFM does not make use of a number of hypotheses which form the basis of the traditional one-particle methods. Buryachenko (2007a) demonstrated that the MEFM includes in particular cases the well-known methods of mechanics of strongly heterogeneous media (such as the effective medium and the mean field methods). However, a fundamental feature is that all these indicated methods (effective medium, mean field method, MEFM and others) are based on the same so-called "effective field hypothesis" (EFH) **H1**, according to which each inclusion has an ellipsoidal shape and is located in some effective field, which is homogeneous over the considered inclusion. However, the hypothesis **H1** is merely a zero-order approximation of binary interacting inclusions that results in a significant shortcoming of the MEFM. Exploiting the new proposed background of micromechanics instead of the old one allows Buryachenko (2010a), and Buryachenko and Brun (2011a) to abandon the use of the central concept of classical micromechanics such as effective field

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