International Journal of Solids and Structures 48 (2011) 3471–3483

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Stretch-induced stress patterns and wrinkles in hyperelastic thin sheets

Vishal Nayyar, K. Ravi-Chandar, Rui Huang *

Department of Aerospace Engineering and Engineering Mechanics, The University of Texas at Austin, Austin, TX 78712, United States

ARTICLE INFO

Article history:
Received 1 July 2011
Received in revised form 1 September 2011
Available online 10 September 2011

Keywords:
Buckling
Wrinkle
Thin sheet
Hyperelastic material

ABSTRACT

Wrinkles are commonly observed in stretched thin sheets and membranes. This paper presents a numerical study on stretch-induced wrinkling of hyperelastic thin sheets based on nonlinear finite element analyses. The model problem is set up for uniaxial stretching of a rectangular sheet with two clamped ends and two free edges. A two-dimensional stress analysis is performed first under the plane-stress condition to determine stretch-induced stress distribution patterns in the elastic sheets, assuming no wrinkles. As a prerequisite for wrinkling, development of compressive stresses in the transverse direction is found to depend on both the length-to-width aspect ratio of the sheet and the applied tensile strain in the longitudinal direction. A phase diagram is constructed with four different distribution patterns of the stretch-induced compressive stresses, spanning a wide range of aspect ratio and tensile strain. Next, an eigenvalue analysis is performed to find the potential buckling modes of the elastic sheet under the prescribed boundary conditions. Finally, a nonlinear post-buckling analysis is performed to show evolution of stretch-induced wrinkles. In addition to the aspect ratio and tensile strain, it is found that the critical condition for wrinkling and the post-buckling behavior both depend sensitively on the sheet thickness. In general, wrinkles form only when both the magnitude and the distribution area of the compressive stresses are sufficiently large. The wrinkle wavelength decreases with increasing strain, in good agreement with the prediction by a scaling analysis. However, as the tensile strain increases, the wrinkle amplitude first increases and then decreases, eventually flattened beyond a moderately large critical strain, in contrast to the scaling analysis.

1. Introduction

Thin sheets and membrane structures are used widely in space applications such as inflatable space antennas, solar sails, and radars (Talley et al., 2002; Sleight et al., 2005; Sakamoto and Park, 2005; Wang et al., 2007). Similar structures have also found applications in areas of solar energy systems (Peypoudat et al., 2005) and large-area flexible electronics (Rogers et al., 2001). The main advantage of using these structures in the space applications is due to their lightweight and low space requirement. Among others, surface flatness over a large area is one of the key requirements for many applications using the flexible thin structures (Wang et al., 2007). For instance, in a solar sail, surface wrinkles may lead to problems such as non-uniform sail loading, loss of momentum transfer to sail, and undesirable torques on the spacecraft (Talley et al., 2002). Typically, wrinkles form as a result of structural instability under compressive stresses. However, previous studies have shown that wrinkles often appear in thin sheets under a variety of loading conditions (Jenkins et al., 1998; Su et al., 2003; Leifer and Belvin, 2003; Wong and Pellegrino, 2006a). It is thus important to understand the mechanics of wrinkling for practical applications that require reliable control of surface wrinkles.

Two approaches have been commonly used for wrinkling analysis of elastic membranes: the tension field theory and the bifurcation analysis. In the tension field theory, the membrane is assumed to have zero bending stiffness. This approach was first applied by Wagner (1929) to estimate the maximum shear load that can be carried by a thin web. Stein and Hedgepeth (1961) adopted the approach in analysis of partly wrinkled membranes, where a wrinkling region is assumed whenever one of the in-plane principal stresses becomes negative. Subsequently, the tension field theory has been continuously developed and extended for various applications (e.g., Mansfield, 1970; Danielson and Natarajan, 1975; Wu, 1978; Pipkin, 1986; Steigmann, 1990; Alder et al., 2000; Liu et al., 2001; Coman, 2007). The tension field theory approach typically provides a satisfactory prediction of the stress distribution and wrinkling regions. However, it does not provide detailed information about the wrinkles such as amplitude and wavelength.

In the bifurcation analysis, the membrane is treated as a thin shell with non-zero bending stiffness. Typically, a geometrically nonlinear finite element method is employed using shell elements for numerical analysis (e.g., Tomita and Shindo, 1988; Friedl et al.,

---

* Corresponding author.
E-mail address: ruihuang@mail.utexas.edu (R. Huang).

0020-7683/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijsolstr.2011.09.004