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Modelling of localization and propagation of phase transformation in superelastic SMA by a gradient nonlocal approach

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ABSTRACT

In this work, a nonlocal phenomenological behavior model is proposed in order to describe the localization and propagation of stress-induced martensite transformation in shape memory alloy (SMA) wires and thin films. It is a nonlocal extension of an existing local model that was derived from a micromechanical-inspired Gibbs free energy expression. The proposed model uses, besides the local field of the internal variable, namely the martensite volume fraction, a nonlocal counterpart. This latter acts as an additional degree of freedom, which is determined by solving an additional partial differential equation (PDE), derived so as to be equivalent to the integral definition of a nonlocal quantity. This PDE involves an internal length parameter, dictating the global scale at which the nonlocal interactions of the underlying micromechanisms are manifested during phase transformation. Moreover, to account for the unstable softening behavior, the transformation yield force parameter is considered as a gradually decreasing function of the martensite fraction. Possible material and geometric imperfections that are responsible for localization initiation are also considered in this analysis. The obtained constitutive equations are implemented in the $\text{Abaqus}^{\circledast}$ finite element code in one and two dimensions. This requires the development of specific finite elements having the nonlocal volume fraction variable as an additional degree of freedom. This implementation is achieved through the UEL user's subroutine. The effect of martensitic localization on the superelastic global behavior of SMA wire and holed thin plate, subjected to tension loading, is analyzed. Numerical results show that the developed tool correctly captures the commonly observed unstable superelastic behavior characterized by nucleation and propagation of martensitic phase. In particular, they show the influence of the internal length parameter, appearing in the nonlocal model, on the size of the localization area and the stress nucleation peak.

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1. Introduction

The specific behavior of SMAs on the shape memory effect and pseudoelasticity at one hand and their high mechanical work/volume ratio on the other hand make them particularly well adapted for the design of microcomponents, (Reynaerts et al., 1997; Benard et al., 1998; Tabib-Azar et al., 1999; Kyung et al., 2008; Bellouard, 2008). An example of SMAs whose practical use is rapidly growing is the NiTi, due to its excellent structural properties. The development of finite element tools dedicated for the numerical design of any systems requires the modelling of the material behavior. In the case of small sized SMA samples such as wires and thin films, the material exhibits unstable softening behavior during stress-induced martensite phase transformation. This unstable behavior manifests itself by strain (martensite fraction) localization and propagation, characterized by a stress peak (nucleation) followed by a plateau (propagation). This Luders-like deformation has been

⇑ Corresponding author. E-mail address: Mohamed.Haboussi@ensem.inpl-nancy.fr (M. Haboussi). shown in many experimental works since the one of Shaw and Kyriakides (1995) (see Fig. 1 for a schematic of this behavior). This particular behavior is the consequence of a material-level instability of the phase transformation at the microscale, it is also influenced by geometric factors such as wire thinning (Shaw and Kyriakides, 1997), the radius/length ratio in the case of a tube specimen (He and Sun, 2009a,b; Cai and Dai, 2006), or the width/length ratio for plate (He and Sun, 2010).

Various models are available in the literature, that are dedicated to the macroscopic description of the thermomechanical behavior of bulk SMAs. Among these models, we mention those reported in Tanaka (1986), Tanaka et al. (1995), Raniecki and Lexcellent (1998), Bekker and Brinson (1998), Thamburaja (2005), Panico and Brinson (2007), Popov and Lagoudas (2007), Zaki and Moumni (2007), Peultier et al. (2006), Saint-Sulpice et al. (2009), Arghavani et al. (2010). These models developed within a local context (based on the assumption of a uniform Representative Elementary Volume (REV)) fail to describe the aforementioned softening and localization phenomena. In fact, the material response predicted on the basis of such modelling will suffer from the pathological problem

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