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# Mechanics of interface failure in the trilayer elastic composite

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### ABSTRACT

An exact analysis of the mechanics of interface failure is presented for a trilayer composite system consisting of geometrically and materially distinct linear elastic layers separated by straight nonlinear, uniform and nonuniform decohesive interfaces. The technical significance of this system stems from its utility in representing two slabs joined together by a third adhesive layer whose thickness cannot be neglected. The formulation, based on exact infinitesimal strain elasticity solutions for rectangular domains, employs a methodology recently developed by the authors to investigate both solitary defect as well as multiple defect interaction problems in layered systems under arbitrary loading. Interfacial integral equations, governing the normal and tangential displacement jump components at the interfaces, are solved for the uniformly loaded trilayer system. Interfacial defects, taken in the form of interface strengths. They are examined in a variety of configurations chosen so as to shed light on the various interfacial failure mechanisms active in layered systems.

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#### 1. Introduction

The mechanics of layered composite systems has been a subject of interest to the structures and applied mechanics communities in recent years owing to a diversity of well known applications of these systems in industry (adhesive and protective coatings Chvedov and Jones, 2004; Graziano, 2000; Boelen et al., 2004), health care (dental restorations consisting of ceramic, ceramic filled polymer and cementitious layers; Niu et al., 2008) and structural rehabilitation (adhesion of fiber reinforced plastic plate to damaged concrete beams). In the last application, two layers or slabs of differing material properties are adhered to each other by an adhesive layer assumed to be of negligible dimensions (Teng et al., 2003; Carpinteri et al., 2007; Wang, 2007; De Lorenzis and Zavarise, 2008) or non negligible dimensions (Rabinovitch and Frostig, 2001; Au and Buyukozturk, 2006; Leung and Tung, 2006; Pan and Leung, 2007; Yuan et al., 2007; Rabinovitch, 2008). The fundamental mechanics by which layered systems fail is an interesting problem in its own right, the earliest work (e.g., Ungsuwarungsri and Knauss, 1987) predating the applications cited above by about 20 years. Classical approaches taken in the treatment of this problem are well known and have assumed a decohesive interface utilizing beam theory (e.g., Ungsuwarungsri and Knauss, 1987; Leung and Tung, 2006; Pan and Leung, 2007; Wang, 2007; Carpinteri et al., 2008; Rabinovitch, 2008) or the finite element method (Carpinteri et al., 2007; De Lorenzis and Zavarise, 2008). Linear elastic fracture mechanics has also been used to analyze a sharp interface crack (e.g., Hutchinson and Suo, 1992; Rabinovitch and Frostig, 2001; Au and Buyukozturk, 2006).

Recently a new approach has been developed to treat layered composites, separated by decohesive interfaces, which involves writing exact elasticity solutions for the boundary displacement components for each layer and piecing them together to form integral equations governing displacement discontinuity components normal and tangent to the interface (Nguyen and Levy, 2009). The equations are necessarily nonlinear owing to nonlinear interface traction-separation/slip relations required to characterize the interface. The solution process proceeds by reducing the integral equations to an infinite set of nonlinear algebraic equations which are then truncated and solved numerically. The efficacy of this approach is that: (i) it lacks the conceptual limitations of Euler-Bernoulli beam theory, (ii) it enables the determination of interface separation/slip behavior without solving for the detailed elastic fields within the layers and (iii) it approaches the generality of finite element analysis (FEA) for the class of systems considered, i.e., linear elastic layers. In other words, the methodology applies to an arbitrary number of geometrically and materially distinct layers separated by uniform or nonuniform interface traction-separation/ slip relations. Note that by uniform interface traction-separation/ slip relation we mean a vector valued expression generally dependent on an interface coordinate dependent displacement jump vector. A well known example (Ferrante et al., 1982), which does not allow for interfacial shear, is the relation given by

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