



A comprehensive dynamic model for class-1 tensegrity systems based on quaternions

Massimo Cefalo^{a,*}, Josep M. Mirats-Tur^{b,1}

^a Institut de Robòtica i Informàtica Industrial, Llorens i Artigas 4-6 2nd Floor, 08028 Barcelona, Spain

^b Cetaqua, Passeig dels Til·lers, 3, 08034 Barcelona, Spain

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ABSTRACT

In this paper we propose a new dynamic model, based on quaternions, for tensegrity systems of class-1. Quaternions are used to represent orientations of a rigid body in the 3-dimensional space eliminating the problem of singularities. Moreover, the equations based on quaternions allow to perform more precise calculations and simulations because they do not use trigonometric functions for the representation of angles. We present a thorough introduction of tensegrities and the current state of research. We also introduce the quaternions and provide in the appendix some important details and useful properties. Applying the Euler–Lagrange approach we derive a comprehensive dynamic model, first for a simple rigid bar in the space and, at last, for a class-1 tensegrity system. We present two model forms: a matrix and a vectorial form. The first more compact and easier to write, the latter more suitable to apply the tools and the theory based on vector fields.

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1. Introduction

Tensegrities are mechanical systems born in the art community in the early 60s (Snelson, 1965), at the beginning studied by architects and successively widely used in engineering applications. Among several definitions we can say that a tensegrity system is an aggregation of mechanical elements, carrying either compression or tension but not both, for which at least one equilibrium state exists. Mixing together the terms *tensile* and *integrity*, which emphasize the main characteristic of such structures, the name *tensegrity* was coined by Buckminster Fuller in the early 60s (Fuller, 1962). An interesting and quite general definition was given by Pugh (1976): “A tensegrity system is established when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space”.

Elements working under compression are called *struts* while the elements working in tension are called *tensors*. Usually the struts are rigid bars and the tensors are cables or springs, but several variations may be allowed. Actuating bars and cables allow to dynamically modify their lengths and, hence, change the configuration of the system.

To build a tensegrity structure, several approaches have been proposed. Between others, one of the most powerful techniques

is based on the construction of a base module and of a design pattern. The overall structure is the result of assembling the base module along the design pattern following proper rules to ensure the existence of, at least, one equilibrium point. So, usually, the most famous tensegrity structures have some degree of geometric symmetry in their equilibrium positions.

These systems have a lot of interesting properties. It has been shown, for example, that carefully designing the appropriate net of connections between rigid elements, it is possible to provide to the structure the desired rigidity (within the limits of the material employed). Compared with traditional structures, tensegrities can be much more stiff, much more light and occupy much less space and volume. The main applications in the robotic field, tend to exploit the capability of such systems to be extensible and redundant (in case of one, or more, struts or tensors fail, it is possible to guarantee the functionality of the system by means of other elements). See, for instance, Aldrich (2004), Paul et al. (2006), Masic and Skelton (2004) and Mirats-Tur (2010) for some application concerning manipulators and mobile robotics.

For several years they have been studied only from a static point of view, mainly because there were no possibilities to face with the enormous quantity of computations required to model their dynamics. The static analysis of tensegrity has reached a certain level of maturity, with lots of contributions by different authors and fields of study. See for instance Roth and Whiteley (1981), Connelly (1999) for a mathematic perspective, Calladine and Pellegrino (1992), Motro et al. (1986), Hanaor (1988) for studies about the self-stress states of the structure, or Tarnai (1989), Vassart et al.

* Corresponding author. Tel.: +39 0649766812; fax: +39 064957647.

E-mail addresses: cefalo.m@gmail.com (M. Cefalo), jmirats@cetaqua.com (J.M. Mirats-Tur).

¹ Tel.: +34 933124879; fax: +34 933124801.