## International Journal of Solids and Structures 48 (2011) 2668-2680

Contents lists available at ScienceDirect



International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr



# Homogenization of aligned "fuzzy fiber" composites

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# ARTICLE INFO

#### ABSTRACT

Article history: Received 2 February 2011 Received in revised form 13 May 2011 Available online 26 May 2011

Keywords: Carbon fibers Carbon nanotubes Asymptotic expansion homogenization method The aim of this work is to study composites in which carbon fibers coated with radially aligned carbon nanotubes are embedded in a matrix. The effective properties of these composites are identified using the asymptotic expansion homogenization method in two steps. Homogenization is performed in different coordinate systems, the cylindrical and the Cartesian, and a numerical example are presented. © 2011 Elsevier Ltd. All rights reserved.

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### 1. Introduction

Despite their recent discovery by lijima (1991), the carbon nanotubes (CNTs) have attracted considerable research attention. Nowadays a large variety of composites containing CNTs have been manufactured (Milo et al., 1999; Peigney et al., 2000; Potschke et al., 2004; Wagner et al., 1998; Lourie and Wagner, 1998; Star et al., 2001; McCarthy et al., 2002; Zhu et al., 2003). This scientific interest is derived from the CNTs exceptional properties. Carbon nanotubes are reported to have an axial Young's modulus in the range of 300–1000 GPa, up to five times the stiffness and with half the density of SiC fibers, while their theoretical elongation to break reaches 30–40% (Yakobson and Smalley, 1997; Yakobson et al., 1997; Yu et al., 2000; Wang et al., 2001; Salvetat-Delmotte and Rubio, 2002; Fisher et al., 2002; Popov, 2004).

Modeling of composites containing CNTs has also grown significantly in recent years. The mechanical response in tension of a single CNT embedded in polymer via finite element analysis was studied by Liu and Chen (2003a), while (Odegard et al., 2003) modeled aligned and misaligned CNT composites using the equivalent continuum method in conjunction with the Mori–Tanaka micromechanics method. Fisher et al. (2002, 2003) studied the effects of nanotube waviness on the effective composite properties using finite element analysis and the micromechanics Mori–Tanaka method. Hadjiev et al. (2006) considered buckling of CNTs within an epoxy matrix. Several efforts in CNT composite modeling have focused on the inclusion of less than ideal CNT adhesion to the matrix (Wagner, 2002; Frankland et al., 2003; Griebel and

\* Corresponding author. *E-mail address:* lagoudas@tamu.edu (D.C. Lagoudas). Hamaekers, 2004). The clustering of CNTs in the polymer matrix was studied in Seidel and Lagoudas (2006). In Spanos and Kontsos (2008) nanocomposite properties were computed using Monte Carlo finite element method. Molecular dynamics (MD) simulations have been used to obtain the stress-strain behavior of CNTs embedded in a polymer matrix (Frankland et al., 2002), or the properties of the interphase between CNTs and polymer (Awasthi et al., 2009). In all these modeling efforts, the carbon nanotubes are embedded directly in a polymer matrix.

In this work we focus on composites containing carbon fibers. which are coated with carbon nanotubes ("fuzzy fibers"). In order to achieve significant enhancement of the mechanical behavior of the interface between the carbon fiber and the matrix, we choose to work with "fuzzy fibers" where the CNTs are radially aligned on the surface of the carbon fibers. In these composites, the interphase layer between the fiber and the matrix can be seen as a separate composite material consisting of CNTs in radial arrangement inside a matrix. In this perspective, the "fuzzy fiber" can be studied as a two concentric cylinders material, the fiber and the interphase layer. The elastic response of homogeneous and non-homogeneous cylinders under different boundary conditions was studied by Chatterjee (1970, 1999a,b), Chen et al. (2000), Tarn and Wang (2001), Tarn (2002), Ruhi et al. (2005), Hosseini Kordkheili and Naghdabadi (2007), Chatzigeorgiou et al. (2008), Tsukrov and Drach (2010), Nie and Batra (2010a,b,c).

Our goal is to obtain the effective mechanical properties of unidirectional "fuzzy fiber" composites. In order to achieve it we use a multiscale approach based on the asymptotic expansion homogenization method (AEH). The AEH method is well documented in the literature for periodic composites (Sanchez-Palencia, 1978; Bensoussan et al., 1978; Kalamkarov and Kolpakov, 1997;