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Gradient elasticity in statics and dynamics: An overview of formulations, length scale identification procedures, finite element implementations and new results

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ABSTRACT

In this paper, we discuss various formats of gradient elasticity and their performance in static and dynamic applications. Gradient elasticity theories provide extensions of the classical equations of elasticity with additional higher-order spatial derivatives of strains, stresses and/or accelerations. We focus on the versatile class of gradient elasticity theories whereby the higher-order terms are the Laplacian of the corresponding lower-order terms. One of the challenges of formulating gradient elasticity theories is to keep the number of additional constitutive parameters to a minimum. We start with discussing the general Mindlin theory, that in its most general form has 903 constitutive elastic parameters but which were reduced by Mindlin to three independent material length scales. Further simplifications are often possible. In particular, the Aifantis theory has only one additional parameter in statics and opens up a whole new field of analytical and numerical solution procedures. We also address how this can be extended to dynamics. An overview of length scale identification and quantification procedures is given. Finite element implementations of the most commonly used versions of gradient elasticity are discussed together with the variationally consistent boundary conditions. Details are provided for particular formats of gradient elasticity that can be implemented with simple, linear finite element shape functions. New numerical results show the removal of singularities in statics and dynamics, as well as the size-dependent mechanical response predicted by gradient elasticity.

1. Introduction

Classical continuum solid mechanics theories, such as linear or nonlinear elasticity and plasticity, have been used in a wide range of fundamental problems and applications in civil, chemical, electrical, geological, mechanical and materials engineering, as well as in various fields of physics and life sciences. Even though the scales that these theories were initially designed for were ranging roughly from millimetre to metre, to describe deformation phenomena and processes that could be captured by the naked eye, they were also used in the last century to describe phenomena evolving at atomistic scales (elastic theory of dislocations), earth scales (faults and earthquakes) and astronomic scales (relativistic elastic solids). More recently, observations in advanced optical and electron microscopes have been interpreted by using classical continuum mechanics theory; in the last few years standard elasticity formulae have also been used to characterise deformation behaviour at the nanoscale (e.g. nanotubes or other nanoscale objects).

It is in this regime of micron and nano-scales that experimental evidence and observations with newly developed probes such as nano-indenters and atomic force microscopes have suggested that classical continuum theories do not suffice for an accurate and detailed description of corresponding deformation phenomena. More notably size effects could not be captured by standard elasticity and plasticity theories, even though such effects become dominant as the specimen or component size decreases. Moreover, classical elastic singularities as those emerging during the application of point loads or occurring at dislocation lines and crack tips cannot be removed, and the same is true for discontinuities occurring at interfaces. Another important class of problems that could not be treated with classical theory is when the homogeneous stress–strain curve contains a negative slope regime where strain softening or a phase transformation occurs. This is the case with elastic (twinning, martensitic transformations) and plastic (necking, shear banding) instabilities where classical theory could not provide any information on their evolution and spatio-temporal characteristics.

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