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Orientational anisotropy and plastic spin in finite elasto-plasticity

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ABSTRACT

The present paper deals with the orientational anisotropy, in the multiplicative elasto-plastic models with non-zero spin and initial orthotropic anisotropy, under the supposition of small elastic strains, while elastic rotations and plastic deformations are large. A new rate form of the model is derived in the Eulerian setting. The evolution in time for the Cauchy stress, plastic part of deformation, tensorial hardening variables and elastic rotations involves the objective derivatives associated with the same elastic spin. A common plastic spin is allowed in the model as direct consequences that follows from the adopted constitutive framework of finite elasto-plastic materials with isoclinic configurations and internal variables. In this model the orientation of the orthotropy directions are characterized in terms of the Euler angles, which replace the elastic rotations. We provided a constitutive framework valuable for the description of the evolution of the orthotropy orientation during a deformation process whose principal directions are different from the orthotropic axes. Only when the plastic spin is non-vanishing, the orientational anisotropy could develop. We proved that only when there exists an initial orthotropic axis which is orthogonal to the sheet, the rotation of the orthotropic axes remains in plane, i.e. in the plane of the sheet, during a plane deformation process. We investigate the effects of three different analytical expressions for the common plastic spin. We make comparisons with the models and the numerical results already provided in the literature.

1. Introduction

The constitutive model is developed in the finite elasto-plasticity, based on the multiplicative decomposition of the deformation gradient $\mathbf{F}$ into its elastic and plastic components, denoted by $\mathbf{E}$ and $\mathbf{P}$, respectively:

$$\mathbf{F} = \mathbf{E}\mathbf{P}, \tag{1}$$

under the supposition that the elastic strains are small, while the elastic rotations, $\mathbf{R}^e$, are large. We adopt the point of view developed by Cleja-Ţigoiu (1990), Cleja-Ţigoiu and Soós (1990) to describe elasto-plastic materials with internal variables, with respect to plastically deformed configurations (i.e. the so called *local relaxed isoclinic configurations*), and the material symmetry concept introduced in Cleja-Ţigoiu and Soós (1989, 1990) to characterize the structural anisotropy of materials. The general constitutive framework has been applied to a transversal anisotropic elasto-plastic material in Cleja-Ţigoiu (2000a), and the orthotropic finite elasto-plasticity in Cleja-Ţigoiu (2000b). When we consider that the crystallographic axes are kept constant as directions in all local relaxed configurations, we practically realize the so-called isoclinic configurations in the terminology adopted by Mandel (1972) and Teodosiu (1970).

The model proposed here includes *non-zero plastic spin* and three types of anisotropy, namely initial *structural anisotropy* (the orthotropic one), induced anisotropy of *kinematic hardening* type and orientational anisotropy, which are emphasized as soon as plastic deformations start to develop. As a consequence of the material symmetry assumptions made in Cleja-Ţigoiu and Soós (1990), the constitutive and evolution equations should be *invariant* with respect to the material symmetry group, which characterizes the initial material anisotropy. In our case the initial structural anisotropy is characterized by the *orthotropic symmetry group* which is defined in Liu (1982) by

$$g_6 = \{\mathbf{Q} \in Ort | \mathbf{Q}\mathbf{n}_i = \mathbf{n}_i, \quad \text{or} \quad \mathbf{Q}\mathbf{n}_i = -\mathbf{n}_i, \quad i = 1, 2, 3\}. \tag{2}$$

Only *when the elastic strains are small* during elasto-plastic processes, the deformed anisotropic axis remain orthogonal if they were initially orthogonal. In our case this means $\mathbf{m}_i = \mathbf{R}^e\mathbf{n}_i$.

In this paper we shortly present the model with respect to plastically deformed (i.e. isoclinic) configurations in Section 2. Although the paper deals with small elastic strain while the elastic rotations and plastic deformations are finite, we start from the large deformation formalism since the kinematic relationships that follows from the multiplicative decomposition lied to well defined

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