International Journal of Solids and Structures 48 (2011) 346–356

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Poromechanical response of a finite elastic cylinder under axisymmetric loading

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ARTICLE INFO

Article history:
Received 8 April 2010
Received in revised form 14 September 2010
Available online 1 November 2010

Keywords:
Poroelasticity
Cylinder
Stress analysis
Pore pressure
Rock mechanics

ABSTRACT

Drained or undrained cylindrical specimens under axisymmetric loading are commonly used in laboratory testing of soils and rocks. Poroelastic cylindrical elements are also encountered in applications related to bioengineering and advanced materials. This paper presents an analytical solution for an axisymmetrically-loaded solid poroelastic cylinder of finite length with permeable (drained) or impermeable (undrained) hydraulic boundary conditions. The general solutions are derived by first applying Laplace transforms with respect to the time and then solving the resulting governing equations in terms of Fourier–Bessel series, which involve trigonometric and hyperbolic functions with respect to the *z*-coordinate and Bessel functions with respect to the *r*-coordinate. Several time-dependent boundary-value problems are solved to demonstrate the application of the general solution to practical situations. Accuracy of the numerical solution is confirmed by comparing with the existing solutions for the limiting cases of a finite elastic cylinder and a poroelastic cylinder under plane strain conditions. Selected numerical results are presented for different cylinder aspect ratios, loading and hydraulic boundary conditions to demonstrate the key features of the coupled poroelastic response.

1. Introduction

Stress analysis of finite elastic cylinders has received considerable attention in the past due to its close relevance to geotechnical and rock testing methods such as uni-axial and tri-axial compressive tests, double-punch tests and point load strength tests, etc. In addition, stress analysis of cylinders is also relevant to applications involving biomedical and mechanical engineering. Lekhnitskii (1963) and Vendhan and Archer (1978) presented the early analytical solutions for transversely isotropic elastic cylinders by using the methods of stress functions and displacement functions respectively. Later, Chau and Wei (2000) derived the general solution for an isotropic elastic solid cylinder of finite length subjected to arbitrary surface loading based on two displacement functions. Theoretical models of several engineering tests were also presented by Watanabe (1996), Wei et al. (1999), Chau and Wei (2001) for isotropic cylinders and by Wei and Chau (2002) for transversely isotropic materials.

Geological materials such as soils and rocks, and biological materials are normally two-phase materials consisting of an elastic solid skeleton with voids filled with fluid (e.g., water). Such materials are commonly known as poroelastic materials. The theory of poroelasticity has its origin in the one-dimensional theory of soil consolidation proposed by Terzaghi (1923). Biot (1941) developed a general theory of 3-D consolidation by adopting Terzaghi's concepts. Biot's theory is based on the classical theory of elasticity and Darcy's laws and takes into account the coupling between the solid and fluid stresses and strains. Rice and Cleary (1976) reformulated Biot's theory (1941) in terms of Skempton's pore pressure coefficients (Skempton, 1954) and the undrained Poisson's ratio of the bulk material. A comprehensive review of the development of the theory of poroelasticity is given by Detournay and Cheng (1993).

Biot's theory of poroelasticity has widely been applied to model various problems encountered in soil engineering, rock mechanics, biomedical engineering and energy resources exploration. For examples, Abousleiman et al. (1996), Abousleiman and Cui (1998) presented plane strain poroelastic solutions for infinite cylinders subjected to axial strain and confining pressure. Cui and Abousleiman (2001) developed a general solution based on the generalized plane strain conditions for a poroelastic cylinder under an axial load and confining pressure, and examined the poroelastic effects in rock samples under uni-axial and tri-axial testing conditions. Kanj et al. (2003) presented plane strain solutions for a fully saturated transversely isotropic hollow cylinder under various loading conditions relevant to laboratory testing. Recently, Jourine et al. (2004) proposed a general poroelastic solution for radially symmetric plane strain problems to model laboratory testing of thick-walled hollow cylinders with time-dependent boundary conditions. The above studies assumed plane strain conditions, which

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2010.10.005