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# Thermoelastic problem of steady-state heat flows disturbed by a crack at an arbitrary angle to the graded interfacial zone in bonded materials

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ABSTRACT

Plane thermoelasticity solutions are presented for the problem of a crack in bonded materials with a graded interfacial zone. The interfacial zone is treated as a nonhomogeneous interlayer having spatially varying thermoelastic moduli between dissimilar, homogeneous half-planes. The crack is assumed to exist in one of the half-planes at an arbitrary angle to the graded interfacial zone, disturbing uniform steady-state heat flows. The Fourier integral transform method is employed in conjunction with the coordinate transformations of field variables in the basic thermoelasticity equations. Formulation of the current nonisothermal crack problem lends itself to the derivation of two sets of Cauchy-type singular integral equations for heat conduction and thermal stress analyses. The heat-flux intensity factors and the thermal-stress intensity factors are defined and evaluated in order to quantify the singular characters of temperature gradients and thermal stresses, respectively, in the near-tip region. Numerical results include the variations of such crack-tip field intensity factors versus the crack orientation angle for various combinations of material and geometric parameters of the dissimilar media bonded through the thermoelastically graded interfacial zone. The dependence of the near-tip thermoelastic singular field on the degree of crack-surface partial insulation is also addressed.



## 1. Introduction

The last few decades have witnessed impressive progress in the areas of functionally graded materials, in the light of a number of potential benefits that may stem from the use of such media in a broad range of modern engineering practices, especially in elevated temperature environments. From both the phenomenological and mechanistic viewpoints, this progress can largely be attributed to the tailoring capability to produce gradual variations of thermophysical properties in the spatial domain to accommodate a variety of technological issues (Miyamoto et al., 1999). As a result, the utilization of this new generation of engineered materials in the form of a transitional interlayer in bonded media or as a graded coating deposited on the substrate has become one of the highly innovative and promising applications in coping with various shortcomings that are coupled with the apparent property mismatch inherent in the conventional layered systems (Schulz et al., 2003).

When the damage tolerance is a major concern in structural design with the graded components, the distinct problem area would be to identify crack-tip singularities with the aim of quantifying the effect of material gradations on crack driving forces and other fracture parameters under iso- and nonisothermal loading conditions. A comprehensive review of related earlier studies of focal interest was compiled by Erdogan (1998), underscoring the outstanding features regarding the crack-tip behavior that entails graded, nonhomogeneous properties. The most notable is the near-tip stress field retaining the square-root singularity together with the same angular distributions around the crack tip as those in the homogeneous material, independent of crack orientation, when the spatially varying elastic modulus is continuous and not necessarily differentiable near and at the crack tip. Readers are referred to Eischen (1987) and Jin and Noda (1994a) for the correspondence between the near-tip fields in homogeneous and nonhomogeneous bodies. The standard analysis methodologies can thus be applied to cracks in functionally graded materials such that the influence of material gradations manifests itself through the values of crack driving forces.

Under the isothermal loading condition, a number of additional contributions in the quasi-static crack problems were reported, among others, by Choi (1996, 1997), Paulino et al. (2003), and Chan et al. (2008). In particular, the mixed-mode and anti-plane behavior of a crack at an arbitrary angle to the graded interfacial zone in bonded structures was also investigated by Choi (2001a, 2007a), while the problem of an inclined crack in a graded coating was examined by Long and Delale (2005). Besides, Choi (2001b) tackled the problem of a subsurface crack in a substrate with graded layering under Hertzian contact tractions and Dag and Erdogan (2002) presented the solution for a surface crack in a graded

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