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Anti-Plane stress analysis of orthotropic rectangular planes weakened by multiple defects

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ABSTRACT

The solution of a Volterra type screw dislocation problem in an orthotropic rectangular plane with finite length and width and various boundary conditions is obtained by means of a separation of variables technique. A distributed dislocation method is employed to obtain integral equations of the plane with cracks and cavities under an anti-plane traction. The ensuing equations are of the Cauchy singular type and have been solved numerically. Several examples are presented to demonstrate the applicability of the proposed solution.

1. Introduction

The existence of defects in the form of cracks and cavities in composite materials generate regions with high stress gradient. These regions are often the primary source of failure in mechanical components (such as wedges, rectangular planes, strips, etc.), even under moderate load magnitudes. As a result, stress analysis in early stage of a design should be conducted with an account of these defects. In anti-plane elasticity problems, the use of dislocation methods in obtaining a solution of crack problem in infinite or semi-infinite domains is a common practice. This is because the dislocation solution is a Green's function solution of a given crack problem Hills et al. (1996). There are limited numbers of investigations on anti-plane elasticity problems in finite domains weakened by defects, including the work by Chang (1985), Ma (1988, 1989), Faal et al. (2007), Matbuly and Nassar (2009). Chang (1985) and Ma (1988, 1989) studied the solution of central crack problem in rectangular sheets. Faal et al. (2007) conducted the stress analysis of a finite *wedge* weakened by arbitrarily shaped cavities. In the study by Matbuly and Nassar (2009), the anti-plane stress analysis of two interfacial cracks located between two dissimilar orthotropic rectangular planes under shear loading was conducted. Using finite Fourier transforms, the problem was reduced to a system of singular integral equations with Cauchy type singularities, which were then solved numerically to calculate the stress intensity factor (SIF) at crack tips. The work was considered as the reexamination and further development of the problem studied by Li and Duan (2006) using a single interfacial crack.

Stress analysis of *strip*, with a domain which is the most similar to that of rectangular planes, has also been the subject of earlier investigations. Zhou et al. (1998) investigated the stress field near two collinear cracks perpendicular to the edges of an isotropic strip. The cracks were symmetrical with respect to the centerline of the strip and subjected to an anti-plane traction. Li (2003) obtained a closed-form solution of the aforementioned problem where the medium was considered to be orthotropic. The stress analysis in an isotropic strip weakened by two collinear cracks situated on the strip's centerline under anti-plane shear was carried out by Zhou and Ma (1999). According to the above studies, the application of a given boundary condition during solution results in a set of integral equations that can be solved by Schmidt's method. Wu and Dzenis (2002) obtained a closed-form solution for the mode III stress intensity factor of an interfacial edge crack between two bonded semi-infinite dissimilar elastic strips. In another article, Li (2005) considered an interfacial crack between two bonded dissimilar semi-infinite orthotropic strips where the crack surface was under anti-plane traction. Closed-form stress intensity factors were obtained for the strip with either clamped or traction free boundaries. More recently, the anti-plane deformation of orthotropic strips with multiple defects was studied by Faal et al. (2006). In their work, stress analysis was carried out in an orthotropic strip containing Volterra type screw dislocations. Using the dislocation solution, the integral equations of the strip under anti-plane traction were obtained. Finally, by solving the integral equations, the stress intensity factors in crack tips and hoop stresses on cavity boundaries were obtained.

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