International Journal of Solids and Structures 48 (2011) 2605–2613

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Using M-integral for multi-cracked problems subjected to nonconservative and nonuniform crack surface tractions

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ARTICLE INFO

Article history:
Received 4 August 2010
Received in revised form 11 April 2011
Available online 25 May 2011

Keywords:
Multiple cracks
Crack surface tractions
M-integral
Path-independence
Surface creation energy

ABSTRACT

In this paper, an energy parameter based on the concept of the *M*-integral is proposed for describing the fracture behavior of a multi-cracked solid subjected to nonconservative and nonuniform crack surface tractions. By using the *M*-integral with a suitably chosen closed contour, one can evaluate the 'surface creation energy' (SCE) required for creation of the stressed cracks. Also, it is demonstrated that the property of path-independence holds even under the action of crack surface tractions. Therefore, the singular stress field in the near-tip areas is not directly involved in the calculation so that a complicated finite element model around the crack tips is not required in evaluation of the *M*-integral.

1. Introduction

For engineering structures containing multiple distributed cracks, the local stress state in the near-tip region becomes quite complicated and difficult to describe. In such a case, the use of energy parameters in describing the 'global' fracture state of the multi-cracked fracture state is therefore of practical interest. The well-known energy conservation contour integrals derived from Noether's theorem in plane elasticity include the J_k-, M-, and L-integrals (Knowles and Sternberg, 1972; Budiansky and Rice, 1973). Among them, the J_k-integrals ($k = 1,2$) have widely been used as energy fracture parameters for single-cracked problems. Physically, J_k evaluate the energy release rates related to crack extension in quasi-brittle materials. Nevertheless, the J_k-integrals are not suitable for use in characterizing the multi-cracked energy state due to their 'local' nature associated with a single crack tip.

As to the M-integral, while not as commonly used as J_k, it has been used in problems containing a single crack (e.g., Herrmann and Herrmann, 1981; Eischen and Herrmann, 1987; Seed, 1997). In these applications, the integration contours were delimited in various ways. Such flexibility implies its applicability to fracture analysis for multi-cracked problems. In the last decade, the M-integral has been used for problems containing multiple traction-free cracks for linear elasticity (Chen, 2001; Chen and Lu, 2003, etc.) and hyperelastic materials (Chang and Lin, 2007). An important issue addressed in these works is that, for the condition when the cracks are embedded in an infinite medium and subjected to a uniform far-field loading system, the result of M is independent of the coordinate origin. In such a condition, by suitably choosing an integration contour, the M-integral evaluates twice the surface creation energy (SCE) associated with creation of all the cracks and can be used as an energy fracture parameter.

In engineering applications, nonconservative and nonuniform tractions along the crack surfaces – which may be due to pressurized fluids, contact pressure, and interfacial friction – are of special interest. In this case, formulation of the energy conservation contour integrals needs to be modified. Discussion on this issue for problems concerning a single crack tip has been presented (Chang and Wu, 2001). Nevertheless, more investigations on multi-cracked problems are still in need.

The objective of this paper is to evaluate the SCE associated with creation of multiple cracks in 2D elastic solids. In addition to the external loading system, the study is considered especially for problems subjected to nonconservative and nonuniform crack surface tractions. An energy parameter based on the concept of the M-integral is proposed for this purpose. The integral is shown to be path-independent, even under the action of crack surface tractions. As a consequence, accurate solution can be easily obtained by direct use of numerical schemes such as finite element method. This energy parameter can be used to quantitatively characterize the effect of the crack surface tractions on the mechanical strength of the multi-cracked solids.

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doi:10.1016/j.ijsolstr.2011.05.005