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General solution for Eshelby's problem of 2D arbitrarily shaped piezoelectric inclusions

W.-N. Zou [a,*], Q.-C. He [a,b], Q.-S. Zheng [a,c]

[a] Institute for Advanced Study, Nanchang University, Nanchang 330031, China
[b] Université Paris-Est, Laboratoire de Modélisation et Simulation Multi Echelle, UMR 8208 CNRS, 5 bd Descartes, 77454 Marne-la-Vallée, France
[c] Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China

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ABSTRACT

Eshelby's problem of piezoelectric inclusions arises sometimes in exploiting the electromechanical coupling effect in piezoelectric media. For example, it intervenes in the nanostructure design of strained semiconductor devices involving strain-induced quantum dot (QD) and quantum wire (QWR) growth. Using the extended Stroh formalism, the present work gives a general analytical solution for Eshelby's problem of two-dimensional arbitrarily shaped piezoelectric inclusions. The key step toward obtaining this general solution is the derivation of a simple and compact boundary integral expression for the eigenfunctions in the extended Stroh formalism applied to Eshelby's problem. The simplicity and compactness of the boundary integral expression derived make it much less difficult to analytically tackle Eshelby's piezoelectric problem for a large variety of non-elliptical inclusions. In the present work, explicit analytical solutions are obtained and detailed for all polygonal inclusions and for the inclusions characterized by Jordan's curves and Laurent's polynomials. By considering the piezoelectric material GaAs (110), the analytical solutions provided are illustrated numerically to verify the coincidence between different expressions, and to clarify the jump across the boundary of the inclusion and the singularity around the corner of the inclusion.

1. Introduction

Many semiconductor materials are piezoelectric. The coupling effect between mechanical and electric fields has an important contribution to the electronic and optical properties of semiconductor materials (Pan, 2002a,b). Piezoelectric materials have been widely used as sensors and actuators in intelligent advanced structures. For example, a crucial factor in the study of strained semiconductor quantum devices is the strain-induced quantum dot (QD) and quantum wire (QWR) growth (see, e.g., O'Reilly and Adams, 1994; Nishi et al., 1994; Gosling and Willis, 1995; Park and Chuang, 1998; Davies, 1998; Andreev et al., 1999; Faux and Pearson, 2000; Freund, 2000; Pearson and Faux, 2000; Pan and Yang, 2003; Pan et al., 2005). Along with considerable attention attracted by piezoelectric materials, suitable mathematical modeling becomes important to studying electromechanical behaviors. In particular, Green's function technique has been developed both for the three-dimensional (3D) case (Wang, 1992; Dunn and Taya, 1993; Dunn and Wienecke, 1997; Huang and Kuo, 1997; Kuvshinov, 2008) and for the two-dimensional (2D) case (Ting, 1996; Lu and Williams, 1998; Pan, 2002c). For 2D piezoelectric materials, another remarkable technique is the extended Stroh formalism. Because of its preservation of most essential features of the Stroh formalism, the extended Stroh formalism acts as a very powerful tool for the study of piezoelectricity (Ting, 1996; Yin, 2005; Hwu, 2008). Note that the classical Stroh formalism has also extended to solve some three-dimensional anisotropic problems (Wu, 1998; Barber and Ting, 2007).

Eshelby's piezoelectric inclusion problem includes the well-known Eshelby's elastic inclusion problem as a particular one (Eshelby, 1957), corresponding to an infinite homogeneous piezoelectric medium containing a subdomain ω, called an electroelastic inclusion, over which a uniform eigenstrain and/or eigenelectric field is prescibed (see, e.g., Wang, 1992; Ru, 2000; Pan, 2004). It is known that Eshelby's elastic inclusion problem is of prominent importance to a large variety of mechanical and physical phenomena and plays an important role in particular in micromechanics (see, e.g., Willis, 1981; Mura, 1982; Nemat-Nasser and Hori, 1993). So does Eshelby's piezoelectric inclusion problem for piezoelectric materials. Recently, we have obtained explicit analytical solutions to Eshelby's isotropic elastic and anisotropic thermal inclusion problems for a wide variety of non-elliptical inclusions (Zou et al., 2010a,b). For Eshelby's piezoelectric inclusion problem, most of the existing analytical studies concern elliptical/ellipsoidal shapes (Wang, 1992; Liang et al., 1995; Chung and Ting, 1996;

* Corresponding author. Tel.: +86 791 8305343; mobile: 15970409286.
E-mail address: zouwn@ncu.edu.cn (W.-N. Zou).

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