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Analysis of stress concentrations in plates with rectangular openings by a combined conformal mapping – Finite element approach

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ABSTRACT

Plates with rectangular openings develop stress concentrations under bending. While these stresses can be determined using finite elements, in many problems this would be difficult because a high density mesh would be needed in the neighborhood of every opening corner. In this paper, it is shown how a complex-variable conformal mapping approach can be numerically coupled with the finite element method to analyze these corner stresses. This approach can be used even with relatively coarse meshes where the finite element results, by themselves, do not resolve the stress concentrations. In essence, the method relies on finite element analysis to obtain information on the stress field in a region surrounding the plate opening; this information is subsequently used to set up the parameters of the conformal mapping approach to obtain the near field stresses at the opening corners.

1. Introduction

There are a variety of methods for evaluating stress concentrations in plates due to geometrical irregularities such as openings and cracks. In design it is common to use the stress concentration factor, defined as the ratio of maximum local stress to the nominal body or far-field stress. The stress concentration factor can be evaluated by using computational techniques, elasticity theory and experimental stress analysis such as photoelasticity. Peterson (1961) has provided stress concentration factors for different geometric irregularities under various types of loadings. Wu and Mu (2003) developed simple methods to obtain stress concentration factors around circular holes located in finite anisotropic plates and cylinders due to in-plane loads.

Using edge function analysis, Hafiani and Dwyer (1999) studied the effect of the opening shape and the anisotropy of laminated composite plates under plane stress conditions. Henshaw et al. (1996) used finite element analysis to study composite laminates with multiple openings under in-plane loads to demonstrate the increase in the stress concentration at the corners of an opening when another opening is added in its vicinity. Using integral equations, Hu et al. (1993) studied the interactions between openings and cracks in different domains by decomposing the problem into single hole and crack problems using the superposition principle. Chong and Pinter (1984) used finite element analysis to determine the stress concentration factor around large holes in tensile strips in terms of the size of the openings. Durelli et al. (1970) experimentally evaluated the large strains around elliptical holes and used photoelasticity to determine the stresses at those locations.

The method of complex variables (Muskhelishvili, 1975) provides a powerful approach to solve elasticity problems with geometric irregularities. The method uses a complex representation for the displacement field and maps the physical space of the plate with an irregularity to a unit disk via a conformal transformation. The boundary conditions are then expressed in the unit disk space by contour integrals. Researchers have used this method to solve a wide variety of elasticity problems. Savin (1961) was one of the first to apply this method to the plate problem. He calculated the stress concentrations around openings of different shapes under several types of far-field loads.

More recent work on this problem using complex variables include papers by Ukadgaonker and Rao (2000), Xiwu et al. (1995), Wu and Cheng (1999), Chen and Hsu (1996), Bryukhanova (1967), Wang and Hasebe (2000), Tsukrov and Novak (2002), Datsyshin and Marchenko (1985), Vigdergauz (1993), Exadaktylos et al. (2003), Cherkaev and Grabovsky (1998). Ukadgaonker and Rao (2000) solved the bending problem for laminated composite plates with openings of different shapes and Xiwu et al. (1995) considered a finite laminate with elliptical hole under in-plane extensional and shear loads. Datsyshin and Marchenko (1985) solved for the stress concentration around curvilinear cracks in a half-plane problem by satisfying the boundary conditions numerically using Gauss quadrature formulas. Wu and Cheng (1999)

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