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Statics of elastic cables under 3D point forces

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ABSTRACT

The catenary problem for elastic cables is extended to the case of uniformly distributed loads and point forces however oriented in space. The equilibrium equation is written in vector form and its solution, i.e. the deformed shape of the elastic cable, is obtained in closed form for the cases of uniformly distributed load, one point force and many point forces. The formulation is suitable to solve straightforwardly cable structure problems, as shown in the numerical applications.

1. Introduction

The circumstance that a chord under self-weight is unable to maintain its rectilinear configuration, in spite of the tension which can be applied at its ends, was known already in 1638 by Galileo Galilei; however, in Galilei opinion the configuration of the chord was parabolic, in accordance with the flight path of a projectile (Galilei, 1638). The mathematical treatment of the cable theory began in the latter half of the seventeenth century. The initial problem was to determine the equilibrium position of an inextensible string hanging between two points and subjected to various load conditions. In particular, the *catenary problem* consists in finding the equilibrium shape of the cable under self-weight. The non-parabolic shape of the chord was noted by Jungius in 1669 but he was not able to find out the real mathematical expression of the curve. The problem was proposed by Jakob Bernoulli in the *Acta Eruditorum* (Bernoulli, 1691) and, after this date, was tackled and solved separately by Huygens, via pure geometric considerations, and by Leibniz (Bernoulli, 1692; Leibniz, 1691a,b) and Johann Bernoulli, Jakob's brother, via the integral calculus. All the three solutions were published in the *Acta* (Leibniz, 1691a,b). The name *catenary* it seems was due to Huygens, who used it in 1690 in a letter to Leibniz.

All the mentioned works did not take into account cable extensibility. The Hooke's law was postulated in 1675 as an anagram (Hooke, 1675; Kurrer, 2008) and clearly written in 1678 (Hooke, 1678). After these dates, Bernoulli brothers were the first who formulated the differential equation of equilibrium of the elastic cable, following the law postulated by Hooke. Also Euler contributed to the study on the catenary, after a suggestion of Daniel Bernoulli; he uses the variation calculus (the "method of final causes") and shows that the equilibrium configuration is determined by the lowest position of the barycentre of mass, i.e. by the minimum of potential energy of the gravity forces (Euler, 1744; Timoshenko, 1983). From an engineering point of view, the mathematical theory of cables allowed to define rules based on analytical reasoning for the design of suspension bridges (Navier, 1823).

Although the analytical expression for the solution of the elastic catenary is nowadays well known and represents the basis of classical literature on cable structures (O'Brien, 1967; Irvine, 1981), it seems worthy to extend the standard solution, confined to the case of loads acting on the plane of the cable, to more general load conditions. Indeed, after some centuries the interest in the subject remains still relevant as cables are widely used in structural engineering. Even if a major effort has been devoted by researchers to numerical cable dynamics and computational methods seem to allow the solution of any problem, closed form solutions still preserve an important role as they allow a drastic reduction of computational effort, a deeper understanding of the structural behavior and serve as benchmark solutions for numerical procedures. For static setting, the exact solution for an elastic cable with distributed or concentrated vertical loads has been given in the work by Irvine and Sinclair (1976), about two centuries after the discovery of the static solution under self-weight. If the parabolic approximation can be accurate for low sagged cables, the exact solution must be preferred for more general case (inclined cables and/or spatial load conditions). In addition the computational advantage of using approximate solutions is not an issue with nowadays computers so that, recent papers (Lepidi et al., 2007; Such et al.,

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