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Depth-sensing indentation of a transversely isotropic elastic layer: Second-order asymptotic models for canonical indenters

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ABSTRACT

Simple analytical approximations to the frictionless indentation problem for a transversely isotropic layer are obtained for spherical, conical, and pyramidal indenters as well as for axisymmetric indenters of power-low profile and self-similar non-axisymmetric indenters. These approximations are asymptotically exact in the small-contact limit. The results obtained are validated in the case of an isotropic layer for spherical and conical indenters.

1. Introduction

Depth-sensing indentation tests are widely used for determining mechanical properties of small specimens and thin films (Fischer-Cripps, 2004). Indentation testing has also proved useful for identification of mechanical properties of biological materials such as articular cartilage (Jurvelin et al., 1987; Korhonen et al., 2003). However, while the majority of analytical studies analyzing depth-sensing indentation have focused on isotropic material models, articular cartilage has been represented as a transversely isotropic layered material (Garcia et al., 1998; Wu and Herzog, 2002; Wilson et al., 2005). Recently, Batra and Jiang (2008) have reviewed the current state-of-the-art in analytical approaches to contact problems for anisotropic materials. Using Stroh's formalism, they studied the plane strain indentation of an anisotropic elastic layer. An analytical solution of the contact problem for a for transversely isotropic layer indented by a hemispherically ended punch was obtained by England (1962). Recently, the stress–strain state in a transversely isotropic layer was studied in Cortes and Garcia (2005) and Klindukhov (2009). The stress fields in the hard-coating and the soft-coating isotropic thin-film/substrate systems were analyzed by Li and Chou (1997). In a series of papers, Fabrikant (2006, 2011) has developed an analytical approach to linear (with an a priori fixed contact area) contact problems for a transversely isotropic layered medium. Nevertheless, the lack of analytical closed-form solutions for the load-displacement relationships prevents using these results for routine indentation testing. In recent years, finite element models have been developed for simulations of indentations tests for transversely isotropic materials (Korhonen et al., 2002; Li et al., 2009).

As a first approximation, force–displacement data obtained in the depth-sensing indentation tests performed with a spherical indenter are analyzed by fitting them with the Hertzian model of elastic contact, which is based on the so-called assumption of infinite sample thickness (Johnson, 1985). When the sample thickness is finite with respect to the radius of the contact area, it is well known that applications of the Hertzian model lead to systematic errors due to the infinite sample thickness assumption. The case of an isotropic elastic layer of finite thickness deposited on a rigid substrate was studied for several decades (Lebedev and Ufliand, 1958; Vorovich and Ustinov, 1959; Keer, 1964; Hayes et al., 1972; Vorovich et al., 1974; Sakamoto et al., 1996; Argatov, 2001; Dimitriadis et al., 2002). When the substrate stiffness is not high enough to neglect the substrate influence, a more complicated two-layered model for the sample-substrate system should be adopted. This issue has been addressed in the literature (Doerner and Nix, 1986; Xu and Pharr, 2006; Gao et al., 1992), where different finite sample thickness models were developed in the case of isotropic sample material. Very recently (Argatov, 2010), asymptotic models were constructed for frictionless and adhesive (no-slip) indentation of an isotropic elastic layer deposited on an isotropic elastic substrate under the assumption that the contact radius, a, is small compared with the layer thickness, h.

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