



Determination of the length of a short crack at a v-notch from a full field measurement

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ABSTRACT

Determination of the length of a short crack at the root of a v-notch, from a full kinematic field measurement, is performed using a direct method. It is based on a matched asymptotic expansions procedure together with the theory of singularities. The first corrective term of the outer expansion can be straightforwardly expressed as a function of the crack length. Its extraction is achieved through the calculation of the associated generalized stress intensity factors for elastic homogeneous materials as well as bimaterials. Numerical simulations are carried out on a finite element solution disturbed by a random noise. In addition, the method used to compute the generalized stress intensity factors proved accurate and robust.

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1. Introduction

The experiments in fracture mechanics split in two parts, those leading to a brutal crack initiation and the almost instantaneous final rupture of the specimen, and those allowing a stable crack growth: Brazilian test, compression on a drilled plate, DCB test, fatigue loadings. The first group deals mainly with dynamic fracture and is not considered herein. The second group requires an accurate knowledge of the location of the crack tip under consideration. In general, direct visual analysis is largely insufficient and one should make use of optical devices and appropriate analysis to achieve an accurate determination. One of the most recent of them is the digital image correlation (DIC) (see the review by [Hild and Roux \(2006\)](#) and the references quoted there). It rebuilds a 2D displacement field on the surface of a sample from the comparison of two images obtained by a camera. It was originally developed to identify the elastic properties and further constitutive laws and is now extended to the characterization of the stress field around a crack tip through the determination of the stress intensity factors ([Réthoré et al., 2008](#)). The measured displacement field is decomposed onto an enriched finite element basis including discontinuous and singular functions derived from the Williams' expansion around the crack tip ([Moës et al., 1999](#)). Then the stress intensity factors are extracted using an interaction integral. This approach is obviously limited to pre-existing cracks.

In this study we are more interested in the detection of the onset of new cracks especially at the root of v-notches in structures subjected to fatigue loading. The growth of a pre-existing crack is

then a special case where the notch opening is zero. The method proposed herein is based on the use of matched asymptotic expansions ([Cole and Kevorkian, 1980](#)) where the crack length is the small parameter, together with the theory of singularities ([Leguillon and Sanchez-Palencia, 1987](#)). The leading term of the outer expansion correspond to the initial state when there is no crack or no crack extension. Its behavior near the root of the notch involves a singular contribution expressed in terms of power of the distance to the root. The next term, the first correction taking into account the presence of a short crack, can be straightforwardly expressed as a function of the crack length and the dual mode to the already mentioned primary one (the exponents are of opposite sign). An accurate determination of this term allows extracting the corresponding crack length. In addition, the method used to compute the generalized stress intensity factors ([Leguillon and Sanchez-Palencia, 1987](#); [Labossiere and Dunn, 1999](#)) proved accurate and robust. It is based on path independent integrals where the dual mode plays the role of the extraction function.

The same ingredients (singular and dual modes so-called “super-singular”) were used in a similar approach recently proposed to describe the effects of the shift in the position of a crack tip and the consequences of the presence of a small process (plastic) zone ([Henninger et al., 2010](#)). But obviously, again only the case of pre-existing cracks was taken into account.

2. Crack at a corner in a homogeneous material

For simplicity, we consider a symmetric situation. The 2D domain Ω^l is made of an isotropic homogeneous material. It embeds a re-entrant v-notch with a short crack with length l

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