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Mathematical programming approaches for the safety assessment of semirigid elastoplastic frames

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ABSTRACT

This paper presents two complementary mathematical programming based approaches for the accurate safety assessment of semirigid elastoplastic frames under quasistatic loads. The inelastic behavior of the flexible connections and material plasticity are accommodated through piecewise linearized nonlinear yield surfaces. As is necessary for this class of structures, geometric nonlinearity is taken into account. Moreover, only a 2nd-order geometric approximation is included as this is sufficiently accurate for practical structures. The work described has a twofold contribution. First, we develop an algorithm that can robustly and efficiently process the complete (path-dependent) nonholonomic response of the structure in a stepwise (path-independent) holonomic fashion. The governing formulation is cast in mixed static-kinematic variables and leads naturally to what is known in the mathematical programming literature as a mixed complementarity problem (MCP). The novelty of the proposed algorithm is that it processes the MCP directly without using some iterative (and often cumbersome) predictor–corrector procedure. Second, in the spirit of simplified analyses, the classical limit analysis approach is extended to compute the limit load multiplier under the simultaneous influence of joint flexibility, material and geometric nonlinearities, and limited ductility. Our formulation is an instance of the challenging class of optimization problems known as a mathematical program with equilibrium constraints (MPEC). Various nonlinear programming based algorithms are proposed to solve the MPEC. Finally, four numerical examples, concerning practical structures and benchmark cases, are provided to illustrate application of the analyses as well as to validate the accuracy and robustness of the proposed schemes.

1. Introduction

Steel frames with flexible connections represent a common class of structures for which it is important to assess reliably and efficiently their structural safety. With the rapid advancement of computer technology and the ever increasing utilization of limit state principles, the 2nd-order inelastic analysis of such semirigid elastoplastic structures has received considerable attention (see, e.g. Chen and Zhou, 1987; Lui and Chen, 1988; Anderson and Kavianpour, 1991; Tin-Loi and Misa, 1996).

The focus to date has primarily been on evolutive (step-by-step) analysis to trace the entire displacement response of the structure under a known applied loading regime. The methods used typically rely on some iterative technique, often based on primarily repeated elastic iterates. Such schemes are generally computational expensive, especially when practical large size structures are involved. It is often the case that very small load steps are required to achieve not only convergence but also an accurate solution to the underlying nonlinear problem.

The present work similarly targets the evolutive elastoplastic analyses of semirigid frames under quasistatic loads and a 2nd-order geometric assumption. Moreover, we also propose an extension of the classical limit analysis approach to compute, in a single step, the maximum load that the structure can sustain under the simultaneous influence of joint flexibility, material and geometric nonlinearities, and limited ductility. For these two types of analyses, we develop novel, efficient and robust techniques within a mathematical programming framework. The underlying mathematical structure in both our evolutive and extended limit analysis formulations is in fact a complementarity system (Maier, 1970, 1971). Complementarity defines the typical componentwise condition $w_j\dot{z}_j = 0$, $w_j \geqslant 0$ and $\dot{z}_j \geqslant 0$ for all j. Physically, it represents conditions that describe, for instance, plastic conformity. It is also pertinent to mention that such approaches are in fact quite generic and can be applied, obviously with some formal complications, to more sophisticated finite element models.

The novelty of our step-by-step analysis lies in the fact that it does not require the use of some expensive iterative predictor–

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