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A screw dislocation in a functionally graded material using the translation gauge theory of dislocations

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ABSTRACT

The aim of this paper is to provide new results and insights for a screw dislocation in functionally graded media within the gauge theory of dislocations. We present the equations of motion for dislocations in inhomogeneous media. We specify the equations of motion for a screw dislocation in a functionally graded material. The material properties are assumed to vary exponentially along the *x* and *y*-directions. In the present work we give the analytical gauge field theoretic solution to the problem of a screw dislocation in inhomogeneous media. Using the dislocation gauge approach, rigorous analytical expressions for the elastic distortions, the force stresses, the dislocation density and the pseudomoment stresses are obtained depending on the moduli of gradation and an effective intrinsic length scale characteristic for the functionally graded material under consideration.

1. Introduction

Nonhomogeneous media, multilayered structures and functionally graded materials (FGMs) are of considerable technical and engineering importance as well as of mathematical interest (see, e.g., Erdogan (1995)). Generally, such materials refer to heterogenous composite materials in which the material moduli vary smoothly and continuously from point to point. Typical examples of FGMs are ceramic/ceramic and metal/ceramic systems. Compressive reviews on aspects of FGMs can be found in Markworth et al. (1995) and Erdogan (1995). However, only few investigations have been made to assess the role and the importance of dislocations in FGMs. For the first time, Barnett (1972) found the displacement and stress fields of a screw dislocation in an isotropic medium which is arbitrary graded in the *x*-direction. One important example for a FGM are exponentially graded materials. For such media the elasticity moduli are exponential functions depending on the space coordinates and the new material parameters describing the gradation (see, e.g., Erdogan (1995)). The Green function for a two dimensional exponentially graded elastic medium has been found by Chan et al. (2004) and the three-dimensional Green function is given by Martin et al. (2002).

Classical continuum theories possess no intrinsic length scales and, therefore, they are scale-free continuum theories which are not able to describe size effects being of importance at micro- and nanoscales and near defects. In order to describe such effects generalized continuum theories are needed. Such generalized continuum theories are, for instance, strain gradient theories (Mindlin, 1964; Altan and Aifantis, 1997; Lazar and Maugin, 2005) and the dislocation gauge theory (Kadić and Edelen, 1983; Edelen and Lagoudas, 1988; Lazar, 2002; Lazar and Anastassiadis, 2009) which enrich the classical theories with additional material lengths. Using strain gradient elasticity, cracks in FGMs have been investigated by Paulino et al. (2003) and Chan et al. (2008). For the first time ever, the solution of a screw dislocation in FGMs, using the theory of gradient elasticity, was given by Lazar (2007). Gradient elasticity is a theory which is very popular in engineering sciences. But not everything is well-understood about the physical importance of the higher-order stresses (hyperstresses). Gradient elasticity should be understood as an effective theory since it was invented for compatible deformations. On the other hand, a more physically motivated 'gradient-like' theory is the so-called dislocation gauge theory. In the dislocation gauge theory both the incompatible elastic distortion tensor and the dislocation density tensor are physical state quantities giving contribution to the elastic stored energy density. Also for nonhomogeneous media these physical state quantities are gauge-invariant. In this approach, the higher-order stress is realized as pseudomoment stress which is the response to the presence of dislocations. This pseudomoment stress is related to the moment stress known from generalized elasticities like Cosserat elasticity (see, e.g., Nowacki (1986), Eringen (1999)). Thus, the force stress tensor is asymmetric like in Cosserat elasticity. In general, in gradient elasticity the hyperstress (double stress)

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