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Generalized inner bending continua for linear fiber reinforced materials

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ABSTRACT

This paper deals with the effective behaviour of elastic materials periodically reinforced by linear slender elastic inclusions. Assuming a small scale ratio ε between the cell size and the characteristic size of the macroscopic deformation, the macro-behaviour at the leading order is derived by the homogenization method of periodic media. Different orders of magnitude of the contrast between the shear modulus of the material μ_m and of the reinforcement μ_p are considered.

A contrast μ_m/μ_p of the order of ε^2 leads to a full coupling between the beam behaviour of the inclusions and the elastic behaviour of the matrix. Under transverse motions, the medium behaves *at the leading order* as a generalized continuum that accounts for the inner bending introduced by the reinforcements and the shear of the matrix. Instead of the second degree balance equation of elastic Cauchy continua usually obtained for homogenized composites, the governing equation is of the fourth degree and the description differs from that of a Cosserat media.

This description degenerates into, (i) the usual continua behaviour of elastic composite materials when $O(\mu_m/\mu_p) \geqslant \varepsilon$, (ii) the usual Euler–Bernoulli beam behaviour when $O(\mu_m/\mu_p) \leqslant \varepsilon^3$.

The constitutive parameters are derived and can be computed or estimated from simplified geometries. Simple criteria are given to identify the appropriate model for real reinforcements under given loadings.

1. Introduction

Understanding the behaviour of reinforced materials is of interest in engineering mechanics of hand-made fiber oriented materials encountered in aeronautics (mat of carbon or glass fibers, etc.), in civil engineering (pile foundations massif, embankments of reinforced earth, etc.), or in the domain of natural materials studied in biomechanics (bones, vegetal tissues, etc.).

These materials belong to the wider class of composites media on which numerous studies aim at establishing the relation between the constituents, the local morphology and the global behaviour. This is justified by the fact that phenomena in heterogeneous media can be upscaled and formulated in terms of macroscopic behaviour, provided that the condition of scale separation is fulfilled. This latter condition requires a medium morphology sufficiently regular to be described by a representative elementary volume much smaller in size than the characteristic size of the phenomena (Auriault, 1991). In the literature, these conditions are systematically satisfied, implicitly or explicitly. Among the works on upscaling, let us mention the variational approaches, e.g. Hashin (1983), and the asymptotic methods of homogenization of periodic media (Sanchez-Palencia, 1980). For elastic constituents, the homogenization limited to the leading order proves that the macro-behaviour of composites is that of elastic equivalent Cauchy media where the elastic tensor can be determined as soon as the microstructure is known (Léné, 1978). Descriptions accounting for the higher terms introduce so-called "non-local" correctors (Gambin and Kröner, 1989; Boutin, 1996). The "non-local" denomination expresses that the stress state does not depend only on the strain state in an elementary representative volume but takes into account the strain gradient, or equivalently the strain in the neighbour representative volumes. In this sense the stress–strain relation is "non-local". In those previous works it was shown that the leading order description strictly applies for homogeneous macro-strain, whereas in other cases the effective behaviour involves higher gradients of strain (double gradient in most cases i.e., the curvature). Thus, in the range of loading where homogenization applies, composites appear as Cauchy media with small perturbations induced by the correctors, i.e., "slightly non-local" generalized media. As is the case for Cosserat's media or micromorphic media (Eringen, 1968) the fundamental difference with Cauchy media lies in the existence of an intrinsic finite length, related to the cell size of the composite. Numerous works are devoted to this topic and for a recent revue the reader may refer to Forest (2006).

In above mentioned results, the non-local effects in 3-D composites (made of constituents of properties of the same order) appear as correctors and not at the leading order. Conversely, in 1-D (homogeneous) beam theory, the curvature effect dominates. This leads to think that it should be possible to obtain non-local effects at the leading order in 3-D composite made of soft matrix and stiff parallel

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