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# Nonlinear fracture of 2D magnetoelectroelastic media: Analytical and numerical solutions

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#### ABSTRACT

A strip electric–magnetic polarization saturation (SEMPS) model is developed to study the electric and magnetic yielding effects on a crack in magnetoelectroelastic (MEE) media. In this model, the MEE medium is treated as being mechanically brittle, and electrically and magnetically ductile. Analogously to the classic Dugdale model, the electric and magnetic yielding zones in front of the crack are represented for simplicity by two strips. In the electric yielding strip the electric displacement equals the electric displacement saturation and meanwhile in the magnetic yielding zone the magnetic induction equals the magnetic induction saturation. The nonlinear analytical solution of this SEMPS model of crack in an infinite MEE medium is obtained using an integral equation approach. The equivalence between the proposed SEMPS model and the existing strip electric–magnetic breakdown (SEMB) model is demonstrated.

To analyze the nonlinear fracture problem in the corresponding finite MEE media, the non-linear hybrid extended displacement discontinuity-fundamental solution (NLHEDD-FS) method is modified, and a multiple iteration approach is adapted to determine the electric and magnetic yielding zones. Comparing with the analytical solution, the applicability and effectiveness of the NLHEDD-FS method is verified. Numerical results based on the SEMPS model for a center crack in infinite and finite MEE strip are presented.

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#### 1. Introduction

Magnetoelectroelastic (MEE) materials with enhanced mechanical–electric–magnetic coupling properties have been intensively studied in the past decades. Due to their remarkable properties, they are widely used in engineering as sensors, transducers and actuators (Van Run et al., 1974). However, under the mechanical and/or electric and/or magnetic loading, these materials are usually predisposed to fracture owing to the existing microscopic defects. Therefore, to improve properties of these materials, it is very important to study the fracture formation and development under various load configurations. However, the problems related to fracture in this material system are rather complex both theoretically and experimentally, except for some simple cases (Chung and Ting, 1995; Liu et al., 2001; Song and Sih, 2003; Hou et al., 2003; Qin, 2004; Jiang and Pan, 2004; Zhao et al., 2006; Wang and Mai, 2007; Dong et al., 2008; Sladek et al., 2008; Zhou and Chen, 2008).

As is well-known, many materials have nonlinear properties, and a plastic zone exists near the crack tip because of the stress and strain singularity under applied loadings. As one of the simplest models in elastic-plastic fracture mechanics, Dugdale model (Dugdale, 1960) treated the plastic zone as a strip in which the stress equals the yield stress. For piezoelectric materials, the electric displacement reaches a saturation point when the applied electric field is sufficiently large according to the electric hysteresis as illustrated schematically in Fig. 1(a). Based on this physical feature and the Dugdale model, Gao et al. (1997) proposed the strip polarization saturation (PS) model for explaining the nonlinear fracture behavior of piezoelectric media. In that model, the electric displacement reaches the saturation value in a line segment in front of the crack and the length of the electric yielding zone was found from the boundary condition of the finite electric displacement at the end of the electrical yielding zone. The PS model was subsequently investigated by Ru (1999), Wang (2000), Jeong et al. (2004), Beom et al. (2006) and Loboda et al. (2008, 2010) among others. From the energy point of view, McMeeking (2001) pointed out that the electric displacement would behave like the strain, and the electric field strength like the mechanical strength. Later, Zhang (2004) and Zhang et al. (2005) proposed the dielectric breakdown (DB) model for fracture in piezoelectric media, in which the electric field reaches the critical value in the yielding strip. Although these two models were established based on two different physical pictures, they surprisingly give the same fracture

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