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Equivalent inclusion solution adapted to particle debonding with a non-linear cohesive law

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ABSTRACT

Starting from Eshelby's solution of the equivalent inclusion problem, an approximate solution is proposed in order to model interface debonding of a spherical inhomogeneity isolated in a uniform matrix. Both phases are linear elastic but the interface traction-separation law is non-linear. A semi-analytical incremental model is developed which is suitable for any type of loading. For computational efficiency, the model relies on two simplifying assumptions: (i) the eigenstrain is uniform inside the inhomogeneity and (ii) the interface compliance is averaged over inhomogeneity's surface when computing the average strain within the inhomogeneity. An extensive parametric study is conducted for three loading modes and 144 combinations of non-dimensional parameters. The predictions are assessed against full-field finite element solutions based on two error measures of the mean stress field inside the inhomogeneity. The results show that the mean error value is acceptable in all cases and indicate the parameter ranges for which the model is most accurate.

1. Introduction

Mechanical behavior of inclusion-reinforced composites is affected by the nature of the bond between the constituents (Duan et al., 2007). The perfect bonding assumption, i.e. continuity of the displacement and the traction vectors across the constituents interfaces (Christensen, 1979), is sometimes an inappropriate idealization (Mura and Furuhashi, 1984; Crouch and Mogilevskaya, 2006). Imperfect bonding may result either from the manufacturing process or from damage development when the composite is loaded. For linear elastic materials, it lowers the effective elastic modulus. For example, in high explosive PBX9501, particle debonding may reduce the bulk modulus by more than 40% (Tan et al., 2005). For non-linear (e.g. plastic) material models, debonding between harder and softer phases may be at the origin of micro cracking, shear banding, or ductile damage (Smith et al., 1988). Another type of imperfect bonding is grain boundary sliding in certain polycrystals or in granular media (Mura and Furuhashi, 1984).

When continuity of traction and displacement is fulfilled at the constituents' interfaces, the macroscopic mechanical response of inclusion-reinforced composites may be predicted at low computational cost based on Eshelby's equivalent inclusion method (Mura, 1987; Eshelby, 1957) combined with a mean field homogenization (MFH) scheme (e.g. Hill, 1965; Mori and Tanaka, 1973; Benveniste, 1987; Christensen, 1991). The outcome of such mean-field modeling approaches both at the macroscopic level and at the phase average level is often assessed based on full-field, finite element solutions obtained on single- or multi-inclusion unit cells (e.g. Garboczi and Berryman, 2001).

Several studies in the literature have aimed at reproducing the effect of imperfect bonding conditions between inclusions and matrix on local stress fields (e.g. Mura and Furuhashi, 1984; Ghahremani, 1980a,b; Hashin, 1991a) and on the overall elastic properties of the composite (Tan et al., 2005; Benveniste, 1985; Hashin, 1991b; Qu, 1993b). Mura and Furuhashi (1984), Mura et al. (1985) and Jasiuk et al. (1987) treated the problem of an ellipsoidal inclusion with free slip along the interface, subjected to a uniform eigenstrain. They imposed continuity of normal displacement (and traction) but vanishing shear stress along the interface. The concept of imperfect interface was introduced and related to continuity of traction across the interface and discontinuity of displacement, including a possible displacement jump normal to the interface. Linear traction-separation laws (or linear spring-type cohesive laws) were considered first. Hashin (1991a) has shown that the strain and stress fields inside an inclusion are non-uniform for linear spring imperfect interface. Benveniste (1985) has derived the effective shear modulus of a particulate composite with linear spring interface. Achenbach and Zhu (1989) have studied the effect of linear spring interface parameters on the overall elastic moduli of composite material. Sangani and Mo (1997) used a multipole expansion method to compute the elastic interactions between the spherical particles in an isotropic material. Nie and Basaran

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