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Plane strain pure bending of sheets with damage evolution at large strains

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ABSTRACT

An analysis of plane-strain bending at large strains for the rigid/plastic incompressible material model including arbitrary strain-hardening and damage evolution laws is performed. The fracture criterion is based on a critical value of the damage parameter. Numerical treatment is reduced to the system of two partial differential equations written in characteristic coordinates. The through-thickness distribution of the principal stresses and damage parameter as well as the variation of the bending moment with the radius of curvature of the concave surface are found for Swift's hardening law and one specific damage evolution law. General tendencies in solution behaviour are in agreement with physical expectations.
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1. Introduction

Pure plane-strain bending at large strains is one of the classical problems in plasticity theory. A number of analytical and semi-analytical solutions have been proposed for various rigid- and elastic–plastic models in the literature (Dadras and Majless, 1982; Gao, 1994; Hill, 1950; Lyamina, 2006; Tan et al., 1995; Verguts and Sowerby, 1975; Wang et al., 1993). Individual approaches for solving the boundary value problem dependent of the material model adopted have been developed in these papers. In contrast to these approaches, a unified method for isotropic materials has been proposed in Alexandrov et al. (2006). The only requirement imposed on the system of constitutive equations is that the material is incompressible. The method has been extended to a class of anisotropic materials in Alexandrov and Hwang (2009) and has been successfully used for springback calculation in the case of elastic–plastic non-linear hardening materials in Alexandrov and Hwang (2010). In particular, it has been shown in Alexandrov and Hwang (2009, 2010) that an effect of elasticity at large strains is negligible, even though the distribution of stress is discontinuous in rigid plastic solutions, unless the stage of unloading is of interest. Therefore, a rigid plastic model is adopted in the present paper. A great number of material models have been proposed to account for damage evolution at large strains. One of most important areas of application of such models is ductile fracture prediction in metal forming processes. These models can be divided into three groups. The first group includes uncoupled models in the sense that the damage evolution equation should be solved after the solution to the boundary value problem of plasticity theory is found. A review of models of this group is given in Atkins (1996). Models of the second group are partly coupled in the sense that the damage parameter reduces the yield surface but material is plastically incompressible. The present paper deals with this group of models and a short review of such models is provided below. Finally, models of the third group are fully coupled in the sense that the damage parameter enters all the constitutive equations of the original model of plasticity theory. In particular, the equation of incompressibility is not satisfied in this case. A typical model of this group has been proposed by Gurson (1977) and then modified by Tvergaard and Needleman (1984) among others. Even though models of the third group are most sophisticated, models of the second group result in better predictions for some applications (Hambli, 2001). Also, models of the second group provide reliable predictions of ductile fracture in metal forming (Behrens and Just, 2002). Therefore, the second group of models is considered in the present paper. Since the equation of incompressibility is valid for such models, the approach developed in Alexandrov et al. (2006) can be adopted to study the pure bending process. The main difference between various damage evolution models of the second group is the damage evolution equation. The most widely used damage evolution equation has been proposed by Lemaitre (1985). The original version of this model includes elastic compressibility but its rigid plastic version is also used in application to metal forming processes (Andrade Pires et al., 2003). Other damage evolution equations coupled with rigid plastic models have been proposed, for example, in Bonora (1997), Chandrakanth and Pandey (1993), Hartley et al. (1997), and Tai (1990). The present paper deals with an extension of the approach to analysis of plane-strain pure bending proposed in Alexandrov et al. (2006) to include quite an arbitrary damage evolution equation in the case of rigid-plastic incompressible materials. An advantage of this ap-

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