



## Elastic interaction of partially debonded circular inclusions. II. Application to fibrous composite

V.I. Kushch<sup>a,\*</sup>, S.V. Shmegeera<sup>a</sup>, L. Mishnaevsky Jr.<sup>b</sup>

<sup>a</sup> Institute for Superhard Materials of the National Academy of Sciences, 04074 Kiev, Ukraine

<sup>b</sup> Risø National Laboratory for Sustainable Energy of the Technical University of Denmark, DK-4000 Roskilde, Denmark

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### ABSTRACT

A complete analytical solution has been obtained of the elasticity problem for a plane containing periodically distributed, partially debonded circular inclusions, regarded as the representative unit cell model of fibrous composite with interface damage. The displacement solution is written in terms of periodic complex potentials and extends the approach recently developed by Kushch et al. (2010) to the cell type models. By analytical averaging the local strain and stress fields, the exact formulas for the effective transverse elastic moduli have been derived. A series of the test problems have been solved to check an accuracy and numerical efficiency of the method. An effect of interface crack density on the effective elastic moduli of periodic and random structure FRC with interface damage has been evaluated. The developed approach provides a detailed analysis of the progressive debonding phenomenon including the interface cracks cluster formation, overall stiffness reduction and damage-induced anisotropy of the effective elastic moduli of composite.

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### 1. Introduction

Mechanical behavior of composite materials is significantly affected by the degree of bonding between the constituents. Interfacial debonding in the form of arc crack is the predominant damage mode of unidirectional fiber reinforced composite (FRC) under transverse loading, which results in rapid loss of stiffness and strength. From the viewpoint of damage-tolerant design of composite structures, it is important to find an optimal balance between the strength and stiffness. To achieve this goal, an effect of interface debonding on the performance of composite needs to be fully understood and adequately implemented in the predictive models.

There is a body of publications on stiffness prediction for fibrous composites with interfacial debonding by means of micromechanics. However, the most existing work is based on models of varying degrees of approximation in the treatment of fiber interaction and local stress and strain fields. Among them, we mention the “dilute” model (Ju, 1991; Lee and Simunovic, 2001) where interaction of fibers is neglected. In the “composite cylinder” model used by Teng (1992) and Tandon and Pagano (1996), interaction between the fibers is taken into account in the self-consistent manner. The Mori–Tanaka method-based model by Takahashi and Chou (1988) assumes debonding to take place over the entire interface.

A dual effective-medium and finite-element study was carried out by Zheng et al. (2000) and Zhao and Weng (2002) for a single partially debonded elliptical fiber. In the effective-medium approach, the fictitious perfectly bonded inclusions were used and the Mori–Tanaka theory was applied to compute the effective moduli of composite. To our knowledge, only a few models for the multiple fibers with imperfect interface are available in literature. Wriggers et al. (1998) considered debonding by a contact formulation which can handle adhesional forces up to a prescribed tensile limit on the contact interface. In the work by Ghosh et al. (2000), interfacial debonding is accommodated by cohesive zone model, with the normal and tangential springs tractions expressed in terms of interfacial separation.

In the unit cell approach (Nemat-Nasser et al., 1982; Golovchan et al., 1993; Chen and Papathanasiou, 2004; Kushch et al., 2008; among others), an actual FRC is modeled by the equivalent periodic microstructure with a unit cell containing a certain number of inclusions. This model is advantageous in that it takes into account interactions of a whole infinite array of inhomogeneities whereas its deterministic geometry enables an accurate solution of the model problem. In application to the above problem, only the simplest models of this kind, namely, a composite containing a square or hexagonal array of equally debonded fibers, were considered. In the elastic contact model by Shan and Chou (1995), the fiber–matrix interface is assumed completely debonded. Yuan et al. (1997) simulated the debonded interface by uni- and doubly-symmetric interface cracks. In these papers, the interface bonding conditions are fixed in the problem statement. The interfacial failure option

\* Corresponding author.

E-mail address: [vkushch@bigmir.net](mailto:vkushch@bigmir.net) (V.I. Kushch).