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Spacetime dimensional analysis and self-similar solutions of linear elastodynamics and cohesive dynamic fracture

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ABSTRACT

We present a dimensional analysis and self-similar solutions for linear elastodynamics with extensions to dynamic fracture models based on cohesive traction–separation relations. We formulate the problem using differential forms in spacetime and show that the scaling rules expressed in terms of forms are simpler and more uniform than those obtained for tensor representations of the solution. In the extension to cohesive elastodynamic fracture, we identify and study the influence of certain intrinsic cohesive scales on dynamic fracture behavior and describe a fundamental set of nondimensional groups that uniquely identifies families of self-similar solutions. We present numerical studies of the influence of selected nondimensional parameters on dynamic fracture response to verify the dimensional analysis, including the identification of the fundamental set for cohesive fracture mechanics. We show that distinct values of a widely-used nondimensional quantity can produce self-similar solutions. Therefore, this quantity is not fundamental, and it cannot parameterize dynamic, cohesive-fracture response.

1. Introduction

There are numerous applications of dimensional analysis[1] and similarity methods[2] in the fields of fluid mechanics, thermomechanics, electromagnetics and astronomy by Langhaar (1951), Sedov (1959), Huntley (1967), Isaacson and Isaacson (1975) and Szirtes (1998). Although applications of these methods to solid mechanics exist, they are less common and tend to be more limited in scope. For example, the analyses of elastodynamics in Miles (1960) and Norwood (1973) yield both the similarity variables and the complete similarity solutions, but only for specific planar configurations.

Historically, the application of dimensional analysis to modeling of fatigue and fracture of materials was limited by inadequate knowledge of the significant variables that govern these phenomena (Langhaar, 1951; Wagner, 1984). Nonetheless, several applications to *Linear Elastic Fracture Mechanics* (LEFM) can be found in the literature. Carpinteri (1982) and Wagner (1984) derive complete sets of nondimensional parameters using the Buckingham theorem (Buckingham, 1914), including the familiar Griffith's form in the latter work. Setien and Varona (1996) discuss the computation of stress intensity factors in the context of dimensional analysis, and Szata (2001) derives fatigue crack growth rates in isotropic bodies via the universal graph method, which differs slightly from the Buckingham method.

Progress in understanding the microscopic mechanisms of material failure enable new applications of dimensional analysis. For example, dimensional analysis has been used to determine size effects and the dominant failure modes in fracture. Kysar (2003) considers dislocation-induced deformations to obtain a set of nondimensional parameters that control crack-tip energy dissipation and to identify the dominant failure mode, ductile or brittle, at the onset of crack propagation.

Cohesive models are among the most effective, and currently the most popular, class of continuum numerical models for dynamic fracture. They developed from the cohesive zone models first introduced by Dugdale (1960) and Barenblatt (1962). Cohesive models simulate crack initiation and extension by modeling the macroscopic effects of various nonlinear damage processes in the neighborhood of the crack tip. A constitutive relation, called a *traction–separation relation* (TSR), describes the tractions acting across a cohesive interface as nonlinear, bounded functions of the interface separation.

A limited literature on dimensional analysis of cohesive fracture models exists. Carpinteri (1989, 1991) demonstrated that a nondimensional brittleness factor, obtained from cohesive scales and a

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[1] Dimensional analysis is a method by which we can deduce information about a given phenomenon by assuming only that the phenomenon can be described by dimensionally-consistent relations between a selected set of variables (Langhaar, 1951). It can generate a partial solution to nearly any problem with relatively little effort, even when a complete mathematical formulation of the problem is not available (Wagner, 1984).

[2] Similarity methods attempt to represent families of solutions that share a common form when expressed in terms of certain nondimensional *similarity variables*. Techniques used to identify the similarity variables include dimensional analysis (Birkhoff, 1948), group theory (Morgan, 1952), universal graph methods (Szata, 2001) and integral transforms (Norwood, 1973).

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